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Journal

of

THE ROYAL AERONAUTICAL SOCIETY

MARCH 1953

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Journal of

THE ROYAL AERONAUTICAL SOCIETY

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All communications for publication in, and advertisements for, the Journal should be addressed to the Editor, Joan Bradbrooke, A.R.Ae.S.; all communications on general matters affecting the Society should be addressed to the Secretary, A. M. Ballantyne, T.D., B.Sc., Ph.D., A.F.R.Ae.S.

None of the papers or paragraphs must be taken as expressing the opinion of the Council of the Royal Aeronautical Society unless stated otherwise.

THE ROYAL AERONAUTICAL SOCIETY

Scholarships and Awards

The awards administered by the Society fall into three main groups:—

(*a*) Scholarships and Educational Grants. (*b*) Medals awarded for achievement in the profession of aeronautics. (*c*) Medals and prizes awarded for papers read before, or published by, the Society. In addition to the named awards there are special premiums attached to the Wilbur Wright Memorial Lecture and the British Commonwealth and Empire Lecture. Also, there is an annual sum of £250 from which premium awards are made to authors of those papers published in the JOURNAL which are considered to be of particular merit.

SCHOLARSHIPS

The Charter Scholarship—Awarded annually, or at such other times as the Council may decide, to assist a student wishing to undertake advanced work or study in aeronautics. An extension for a second year is looked upon favourably by the Council if it is clear that the student and the work will benefit. The scholarships are awarded only to those who can satisfy the Council that they have the necessary qualifications to benefit and whose proposals for work or further studies are acceptable to the Council. Normally the work should lead to a higher degree or post-graduate diploma.

The Geoffrey de Havilland Memorial Scholarship—Awarded annually, at the discretion of the Council, to assist those engaged in aeronautical research, either theoretical or practical, to undertake advanced study in the field in which they are engaged. The scholarship may be extended for a second year at the discretion of the Council. Scholarships are awarded only to those who can satisfy the Council that they have the necessary qualifications to benefit and whose proposals for work or further studies are acceptable to the Council. Applications should either be graduates or have reached graduate standard.

The Busk Studentship in Aeronautics—Awarded annually at the discretion of the Council to enable the holder to engage in research or preparation for research in aeronautics of the kind which Edward Tashmaker Busk made his own, namely Stability, Meteorology and Gust Research, Aeronautical Instruments, Sighting and Appliances for Bomb Throwing, the calculations and mathematical problems relating to these subjects and generally, any scientific work in connection with any branch of art or trade which conduces to progress in, or the use of flying. The Studentship is awarded for one year but may be extended at the discretion of the Council. It is open to a British Subject less than 25 years of age.

ACHIEVEMENT IN AERONAUTICS

Society's Gold Medal—The highest honour which the Society can confer for work of an outstanding nature in Aeronautics.

Society's Silver Medal—Awarded for work of an outstanding nature in Aeronautics.

Society's Bronze Medal—Awarded for work leading to an advance in Aeronautics.

British Gold Medal—Awarded for outstanding practical achievement leading to advancement in Aeronautics.

British Silver Medal—Awarded for practical achievement leading to advancement in Aeronautics.

Wakefield Gold Medal—Awarded annually, at the discretion of the Council, to the designer or inventor of any apparatus tending towards safety in flying.

R. P. Alston Memorial Prize—Awarded for practical achievement associated with the flight testing of aircraft.

AWARDS FOR PAPERS

The George Taylor (of Australia) Gold Medal—Awarded annually, at the discretion of the Council, for the most valuable contribution read before, or received by, the Society on Aircraft Design, Manufacture or Operation.

Simms Gold Medal—Awarded annually, at the discretion of the Council, for the most valuable contribution read before, or received by, the Society on any subject allied to aeronautics, such as, structures, meteorology, metrology.

Herbert Akroyd Stuart Memorial Prize—Awarded, at the discretion of the Council, for the most valuable contribution read before, or received by, the Society on Applied Thermodynamics.

Edward Busk Memorial Prize—Awarded annually, at the discretion of the Council, for the most valuable contribution read before, or received by, the Society on Applied Aerodynamics.

Pilcher Memorial Prize—Awarded annually, at the discretion of the Council, for the most valuable paper read by a Graduate or Student during the previous year at any meeting of the Society or its Branches.

Usborne Memorial Prize—Awarded annually, at the discretion of the Council, for the best contribution to the Society's publications written by a Graduate or Student on some subject of a technical nature in connection with Aeronautics.

Orville Wright Prize—This prize is offered annually for the best contribution received for publication in the AERONAUTICAL QUARTERLY of the Society on some subject of a technical nature in connection with Aeronautics.

Branch Prize—The Council offer an annual prize for the best paper read before the Branches during the previous lecture session. The Prize is open to any member of the Society or of any Branch.

Secretary's News Letter

March 1953

LAST year two new branches were formed, Merthyr Tydfil and Henlow, and already this year I am able to announce the formation of the 24th Branch of the Society at Singapore. Squadron Leader A. D. Miller has acted as the interim secretary, and in this office he has been confirmed. Air Marshal R. O. Jones has accepted the invitation to give the inaugural address, which is to be delivered early in March. They are to be congratulated on the speed at which they have begun to function. I know that every Member of the Society will wish them well.

In this JOURNAL there is printed a letter which I received from the Secretary of the New Zealand Branch. It was so interesting to me that I have had it printed for your interest. It seems to me significant that Mr. Coleridge has mentioned that the Society helps to find and cement friendships as well as fulfilling its primary functions of disseminating technical knowledge. It is a music hall joke that where two old boys of the same school foregather inevitably there is the foundation of an Old Boys' Association. It may be in the future when you have aeronautical engineers meeting in like circumstances it will result in the formation of Branches and Divisions of the Royal Aeronautical Society. Shakespeare sums up—"a consummation devoutly to be wished."

Council have agreed that exemptions to the Associate Fellowship Examination be given for academic qualifications obtained at the College of Aeronautics, Cranfield, The Technological Institute, Loughborough College, and the Royal Naval Engineering College, Manadon. These exemptions are for those obtaining the qualifications after 1st January 1953. Certain conditions have been laid down by the Council and accepted by the establishments.

The British Commonwealth and Empire Lecture will be given on Thursday 11th June by Sir Hubert Walker, C.B.E., Chairman of West African Airways Corporation. It will be followed by a reception at 4 Hamilton Place. This will be open to all Members of the Society. Particulars will be given later.

The Garden Party will be held this year at Hatfield on Sunday 14th June. The theme will be "history." I hope that there will be on show, if not flying, some of the aircraft which are important in the long roll of aeroplanes. I shall be very pleased to learn of the whereabouts of any old, or even not-so-old, aircraft, which might otherwise be overlooked.

The Society will celebrate the 50th anniversary of the flight by the Wright Brothers by a Dinner which will be held in the Dorchester Hotel on Thursday 17th December 1953. I hope that the many who have contributed and are contributing to the 50 years of aviation history will be able to be present. Divisions and Branches might also consider the possibility of holding a Jubilee Dinner, perhaps on a Regional basis.

The Annual Dinner of the Portsmouth Branch was held on 27th February. This Branch has been somewhat handicapped by the departure of the design staff of de Havillands (Airspeed) from Portsmouth to Christchurch. Despite this their dinner was very successful, and both the Chairman, Mr. G. H. M. Twyman, and the Secretary, Mr. E. M. Bellamy, are to be congratulated. There may be some members of the Society, living locally, who are not members of the Branch. There are certainly some aeronautical engineers in Portsmouth who are neither members of the Society nor the Branch, and these ought to give their support.

A. M. Ballantyne

Secretary

NOTICES

An annual sum of £250 is available for premium awards, usually 15 guineas each, for papers published in the Journal. Members and non-members of the Society are invited to submit papers on any aspect of aeronautics

Annual General Meeting 7th May 1953

Notice is hereby given that the Annual General Meeting of the Royal Aeronautical Society, with which is incorporated the Institution of Aeronautical Engineers, will be held on Thursday, 7th May 1953, at 5.30 p.m. in the offices of the Society, 4 Hamilton Place, London, W.1.

Agenda

1. To read the Notice convening the Meeting.
2. To receive and deliberate upon the Report of the Council on the state of the Society and the Balance Sheets and Income and Expenditure Accounts of The Royal Aeronautical Society and Aeronautical Trusts Limited for the year ended 31st December 1952.
3. To receive the names of those elected to Council for the years 1953-56.
4. To announce the names of Fellows elected by the Council in accordance with By-Law 4.
5. To elect the Auditors for the year 1953.
6. Any other business.

By Order of the Council.

A. M. Ballantyne,
Secretary.

NOTE—In accordance with the By-Laws any member whose subscription has not been paid before the first day of April is not entitled to vote.

Light refreshments will be served after the meeting.

Easter Holidays

The offices and Library of the Society will be closed from 5 p.m. on Thursday 2nd April, until 9 a.m. on Tuesday 7th April 1953.

New Branch

A Branch of the Society has been formed in Singapore. The address of the Honorary Secretary is:—

Squadron Leader A. D. Miller, R.A.F., A.F.R.Ae.S.,
Honorary Secretary,
Singapore Branch of the Royal Aeronautical Society,
c/o Officers' Mess, Royal Air Force,
Seletar, Singapore 28.

The Forty-First Wilbur Wright Memorial Lecture

The Forty-First Wilbur Wright Memorial Lecture will be given by Professor N. J. Hoff, F.R.Ae.S., F.I.A.S., Professor and Head of the Department of Aeronautical Engineering and Applied Mechanics, of the Polytechnic Institute of Brooklyn, New York, on "Structures." The Lecture will be given on 14th September 1953, on the eve of the Anglo-American Aeronautical Conference (15th-17th September 1953).

1953 Garden Party

This year the Society's Aeronautical Garden Party will be held at Hatfield Aerodrome (by kind permission of the Directors of de Havilland Aircraft Ltd.) on Sunday 14th June 1953 in the afternoon.

Further particulars will be published in the next Journal.

Nomination of Candidates for Council

The following is an extract from the By-Laws:—

"The Twenty-one ordinary members (of the Council) shall be nominated and elected from among the members of the Society. At the date of their election at least ten shall be Fellows, and one at least shall be in each of the following classes: Associate Fellow, Associate and Graduate.

"Of the ordinary members of the Council, that number necessary to create seven vacancies shall retire annually. The retiring members shall be those with the longest service since their last election but they shall be eligible for re-election.

"Nominations of candidates for election to the Council must be received by the Secretary not later than *10th April* in each year and shall include statements in writing by the candidates that they are willing to serve. The nomination forms shall be signed by one proposer and two seconders, all of whom shall be voters."

Nomination forms may be obtained on application to the Secretary.

Sixth Louis Bleriot Lecture

The Sixth Louis Bleriot Lecture will be given by General Guy du Merle, F.R.Ae.S., on "Making Commercial Aircraft Pay." The Lecture will be held at the Institution of Mechanical Engineers, Storey's Gate, St. James's Park on Thursday 19th March 1953 at 6 p.m.

Joint Conference of Aslib and Nato

A Joint Conference of the Aslib Aeronautical Group and the Advisory Group for Aeronautical Research and Development (A.G.A.R.D.) of the North Atlantic Treaty Organisation will be held from Friday 27th March to Tuesday 31st March 1953 at the College of Aeronautics, Cranfield. Among the subjects to be discussed are the Cataloguing of Aerodynamic Data (with special reference to the Netherlands plan set out in N.L.L. Report No. 116) and a common international system for indexing, abstracting and cataloguing all aeronautical reports.

The Conference fee will be £1 5s. 0d. (including preprints memberships of the College club and transport from Bedford railway station) and the charge for accommodation will be £1 5s. 0d. a day. Full particulars may be obtained from Mr. C. W. Cleverdon, College of Aeronautics, Cranfield, Bletchley, Bucks.

Members' New Appointments

H. D. Burke (*Associate*), formerly Technical Sales Representative of the Palmer Tyre Ltd., has recently been appointed Manager, General Aircraft Division, Palmer Aero Sales Department.

Flt./Lt. W. R. Carus (*Associate*) has recently been appointed a member of the flying instructional staff at the Royal Air Force College, Cranwell. He was formerly Personal Staff Officer to the Commandant of the Royal Air Force Flying College, Manby.

Douglas S. Causer (*Associate Fellow*), formerly with British Overseas Airways, has been appointed a partner of Causer and Company, Drawing Office Specialists, London, W.12.

B. F. Dockree (*Associate Fellow*) has been appointed Chief Designer of Delaney Gallay Ltd.

G. R. Edwards (*Fellow*) has recently been appointed a Director of Vickers-Armstrongs Ltd., and General Manager and Chief Engineer, Aircraft Division.

H. H. Gardner (*Fellow*) has been appointed Chief Designer, Guided Weapons, of Vickers-Armstrongs Ltd., Weybridge Works.

V. A. Higgs (*Associate Fellow*) has recently been appointed Manager of the Aero and Auto Equipment Sales Department of the British Thomson-Houston Company.

D. Keith Lucas (*Fellow*), Chief Designer, Short Bros. and Harland Ltd., has been made a Director of the Company.

T. F. C. Lawrence (*Associate Fellow*) has recently been appointed to the High Speed Aerodynamics Laboratory of the Department of Supply at Adelaide.

F. K. Main (*Graduate*) has been appointed Chief Draughtsman with the British Ermeto Corporation Ltd., of Maidenhead.

P. R. Owen (*Associate Fellow*), formerly at R.A.E., has recently taken up the appointment of Director of the Fluid Motion Laboratory, Manchester University.

A. A. Rose (*Associate Fellow*), formerly with Saunders-Roe Ltd., has taken up an appointment on the design staff of A. V. Roe (Canada) Ltd.

B. Stephenson (*Associate Fellow*) has been appointed Chief Designer Aircraft of Vickers-Armstrongs Ltd., Weybridge Works.

Professor Phrixos J. Theodorides (*Associate Fellow*) has taken up a new appointment as Research Professor at the Institute for Fluid Dynamics and Applied Mathematics, University of Maryland.

J. A. C. Williams (*Associate Fellow*), formerly with Guthrie and Co. Ltd., of Singapore, has recently been appointed Senior Lecturer in Management at the Wolverhampton and Staffordshire Technical College.

Elliott Memorial Prize

The Elliott Memorial Prize has been awarded to Corporal Aircraft Apprentice P. E. Newton, of the May 1950 Entry at R.A.F., Halton, who has obtained the highest marks in the General Studies Examination. The prize will be presented at the Graduation Prize Giving on 31st March 1953.

Acknowledgments

The Council acknowledge with thanks the return of back numbers of The Journal from J. R. Bradford, Esq., *Associate*; N. A. Champion, Esq., *Associate Fellow*; and A. J. L. Mitchell, Esq., *Associate Fellow*.

Society for Experimental Stress Analysis

The Spring meeting of the Society for Experimental Stress Analysis will be held at the Hotel Schroeder, Milwaukee, Wisconsin, from 20th to 22nd May 1953. Particulars may be obtained from Mr. W. F. Hofmeister, Metallurgical Laboratory, Chain Belt Company, 4501 West Greenfield Avenue, Milwaukee, Wisconsin.

Associate Fellowship Examination Results

December 1952

The following were successful in the Associate Fellowship Examination held in December 1952:—

Part I—Home

D. G. A. Bridgnell, Properties of Matter, Heat, Light and Sound.

D. R. Brodie, Aerodynamics I.

K. G. Draper, Properties of Matter, Heat, Light and Sound.

J. W. Drife, Pure Mathematics; Properties of Matter, Heat, Light and Sound; Strength of Materials and Theory of Structures I.

D. J. Frater, Properties of Matter, Heat, Light and Sound.

D. R. Henbest, Properties of Matter, Heat, Light and Sound.

J. L. Johnston, Properties of Matter, Heat, Light and Sound; Aerodynamics I; Strength of Materials and Theory of Structures I.

T. C. Lee, Applied Mathematics; Properties of Matter, Heat, Light and Sound; Theory of Machines.

J. MacNaughton, Properties of Matter, Heat, Light and Sound; Strength of Materials and Theory of Structures I; Aerodynamics I.

K. J. Neeves, Applied Mathematics; Magnetism and Electricity; Strength of Materials and Theory of Structures I; Theory of Machines.

K. G. Page, Properties of Matter, Heat, Light and Sound.

N. R. Reddy, Strength of Materials and Theory of Structures I.

D. J. Richardson, Aerodynamics I.

R. D. Saha, Strength of Materials and Theory of Structures I; Theory of Machines.

B. Singh, Strength of Materials and Theory of Structures I.

R. P. Stanford, Pure Mathematics; Aerodynamics I; Strength of Materials and Theory of Structures I.

I. K. Trundley, Pure Mathematics; Properties of Matter, Heat, Light and Sound; Aerodynamics I; Strength of Materials and Theory of Structures I.

P. Shu-Yan Wong, Applied Mathematics; Properties of Matter, Heat, Light and Sound; Strength of Materials and Theory of Structures I; Theory of Machines.

Part I—Abroad

W. K. Aiyadurai, Melbourne, Australia, Pure Mathematics; Properties of Matter, Heat, Light and Sound; Strength of Materials and Theory of Structures I; Theory of Machines.

K. S. Chandrashekaran, Bangalore 4, India, Aerodynamics I; Strength of Materials and Theory of Structures I.

S. Singh, Delhi, India, Strength of Materials and Theory of Structures I; Theory of Prime Movers and Fuels.

A. Subrahamanyan, Bangalore, India, Properties of Matter, Heat, Light and Sound; Theory of Prime Movers and Fuels.

Part II—Home

P. J. Farmer, Aircraft Materials.

D. W. Pickston, Aircraft Materials; Engineering Economics.

C. R. Raynham, Strength of Materials and Theory of Structures II.

D. Rowe, Strength of Materials and Theory of Structures II.

P. Simpson, Aircraft Design.

I. H. Smart, Strength of Materials and Theory of Structures II.

F/Lt. G. Wood, Air Transport; Aircraft Materials.

Part II—Provincial

A. G. Creak, Cirencester, Glos., Aerodynamics II.

W. J. Hanlon, Belfast, N. Ireland, Strength of Materials and Theory of Structures II.

W. F. Meichan, R.A.F. Aldergrove, N. Ireland, Meteorology; Navigation.

Part II—Abroad

A. F. El-Kerdani, Heliopolis, Egypt, Aerodynamics II.

P. Krishna Iyer, Chromepet, Madras, India, Engine Design.

R. B. Lal, S.P.O., Madras, India, Engine Design.

Diary

LONDON

March 11th

GRADUATES' AND STUDENTS' SECTION. Annual General Meeting. 4 Hamilton Place, W.1. 7.30 p.m. Followed by an aeronautical film show, including the "S.B.A.C. Show 1952." Coffee 7-7.30 p.m.

March 12th

MAIN LECTURE at **DERBY**. Some Aspects of High Performance Jet Aircraft. Sqdn. Ldr. W. A. Waterton, G.M., A.F.C. Rolls-Royce Welfare Hall, Derby. 6.15 p.m.

March 17th

GRADUATES' AND STUDENTS' Section. Visits to the "Daily Mail" and "Daily Herald" printing works.

March 19th

SIXTH LOUIS BLERIOT LECTURE. Making Commercial Aircraft Pay. General Guy du Merle. Institute of Mechanical Engineers, Storey's Gate, S.W.1. 6 p.m. (Tea 5.30 p.m.)

March 25th

GRADUATES' AND STUDENTS' SECTION. Convertible Aircraft. J. Shapiro. 4 Hamilton Place, W.1. 7.30 p.m.

March 27th

FULL DAY DISCUSSION ON FATIGUE. At the Chemistry Lecture Theatre, University College, Gower Street, W.C.1. 10 a.m. to 6.30 p.m.

March 31st

SECTION LECTURE. Experimental Work on Boundary Layer Flow. W. E. Gray. In the Library, 4 Hamilton Place, W.1. 7 p.m.

April 22nd

GRADUATES' AND STUDENTS' SECTION. Aircraft Accident Investigation. H. Caplan. In the Library, 4 Hamilton Place, W.1. 7.30 p.m.

April 23rd

MAIN LECTURE at **GLASGOW**. Civil Jet Operations. Captain A. M. Majendie. The Royal Technical College, Glasgow. 7.30 p.m.

BRANCHES

March 9th

Halton—Films. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

March 11th

Brough—Airships. Lord Ventry. Electricity Showrooms, Ferensway, Hull. 7.30 p.m.

Halton—Visit to Vickers-Supermarine, Winchester.

Hatfield—Navigational Problems Associated with the Introduction of Comet Aircraft. Captain A. M. Majendie. Senior Staff Mess, de Havilland Aircraft Co. Ltd., Hatfield. 6.15 p.m.

Southampton—Can We Reach Mars? E. G. Burgess. Institute of Education, University of Southampton. 7 p.m.

March 12th

Derby—MAIN SOCIETY LECTURE. Some Aspects of High Performance Jet Aircraft. Sqdn. Ldr. W. A. Waterton, G.M., A.F.C. Rolls-Royce Welfare Hall. 6.15 p.m.

Isle of Wight—New Materials and Types of Aircraft Structure. H. J. Pollard. Club House, Saunders-Roe Sports and Social Club, Church Path, E. Cowes. 6 p.m.

March 12th

Portsmouth—Films. Technical School of the de Havilland Aircraft Co. (Portsmouth) Ltd., The Airport, Portsmouth. 6 p.m.

March 16th

Halton—Branch Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

Henlow—Helicopter Design Problems. A. H. Yates. 163 Building, R.A.F. Technical College. 7.30 p.m.

March 18th

Coventry—Some Aspects of Airline Flying from the Pilot's Point of View. Captain R. Rymer, B.E.A. Wine Lodge. 7.30 p.m.

Leicester—Helicopter Design Problems. A. H. Yates. Lecture Theatre, College of Technology, Leicester. 7.15 p.m.

Reading and District—Flight of Birds. Captain J. L. Pritchard, C.B.E. Abbey Gateway Rooms, Abotts Walk, Reading. 7.45 p.m.

Weybridge—Making Aircraft Films. P. D. Normanville. Vickers-Armstrongs Ltd., Weybridge Works. 6 p.m.

March 19th

Glasgow—Engine Development Flight Testing. Wing Commander T. H. Heyworth. Rolls-Royce, Hillington. 7.30 p.m.

Southampton—Supper Dance.

March 21st

Birmingham—Annual Dinner. White Horse Hotel, Birmingham.

March 23rd

Bristol—Junior Members' Papers Competition. Conference Room, Filton House, Bristol Aeroplane Co. Ltd. 6 p.m.

Halton—Junior Members' Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

March 25th

Hatfield—Discussion evening for Members' Papers. Senior Staff Mess, de Havilland Aircraft Co. Ltd., Hatfield. 6.15 p.m.

Preston—Some Modern Aspects of Production. Professor J. V. Connolly. The Assembly Hall, Technical College, Corporation Street. 7 p.m.

March 26th

Portsmouth—Annual General Meeting. Technical School of the de Havilland Aircraft Co. (Portsmouth) Ltd., The Airport. 6 p.m.

March 31st

Belfast—Rocket Propulsion and Interplanetary Flight. A. V. Cleaver. New Hall, Kensington Hotel, College Square East, Belfast. 7 p.m.

Henlow—Aerodynamics of Wind Tunnel Design. J. C. Gibbings, 163 Building, R.A.F. Technical College, Henlow. 7.30 p.m.

April 1st

Chester—Fire Prevention in Aircraft. E. J. White. Grosvenor Hotel, Chester. 7.30 p.m.

April 8th

Weybridge—The Work of the Estimating, Planning and Progress Departments. J. F. Hawkins, W. R. Coomber and P. D. Imlach. Vickers-Armstrongs Ltd., Weybridge Works. 6 p.m.

April 13th

Derby—Electrics. H. West. Rolls-Royce Welfare Hall, Nightingale Road, Derby. 6.15 p.m.

Glasgow—Annual General Meeting. St. Enoch Hotel. 7.30 p.m.

April 14th

Bristol—Annual General Meeting and Film Show. Conference Room, Filton House, Bristol Aeroplane Co. Ltd. 6 p.m.

April 15th

Hatfield—Fatigue: What It Is and Ways of Reducing its Incidence. Major P. L. Teed. Senior Staff Mess, de Havilland Aircraft Co. Ltd., Hatfield.

Preston—Annual General Meeting, followed by a Brains Trust. Assembly Hall of the Technical College, Corporation Street, Preston. 7 p.m.

Reading and District—Meteorological Service for B.O.A.C. Comet Operation. E. Chambers. The Abbey Gateway Room, Abotts Walk, Reading. 7.45 p.m.

Southampton—Aircraft Maintenance. B. S. Shenstone. Institute of Education, University of Southampton. 7 p.m.

April 20th

Halton—Branch Night. Branch Hut, R.A.F. Station, Halton. 6.45 p.m.

April 21st

Belfast—Annual General Meeting. New Hall, Kensington Hotel, College Square East, Belfast. 7 p.m.

April 22nd

Coventry—Annual General Meeting and Films on Lubrication, by courtesy of Esso. Wine Lodge. 7.30 p.m.

April 23rd

Glasgow—MAIN LECTURE—Civil Jet Operations. Captain A. M. Majendie. Royal Technical College, Glasgow. 7.30 p.m.

April 27th

Halton—Bird Flight. Captain J. L. Pritchard, C.B.E. Branch Hut. R.A.F. Station, Halton. 6.45 p.m.

Henlow—Student Members' Premium Lectures. R.A.F. Technical College, Henlow. 7.30 p.m.

THE NEW ZEALAND DIVISION

Members may be interested in the following letter to the Secretary from the Honorary Secretary of the New Zealand Division of the Society.

16th December 1952.

Dear Dr. Ballantyne:—

Our report and financial statement will be coming forward early in the New Year and that will detail the facts of our activities for the year. Although the growth of the Division is not shown by the figures as being very rapid, there is an awakening interest in the doings of the Society which seems to be fulfilling a real need in the country. Aviation activities, though not very profound in this country, are on a surprisingly wide basis. We cannot muster large numbers of experts in aeronautical matters, but we do manage to gather together very large numbers (for the size of the country) of people who are extremely interested in aeronautical matters in a general or even an amateur way. A large part of this class of person is young and this is an extremely promising sign to find that the activities of the Society are catering for the rising generation.

The problems of Aviation in this country are without doubt very different from the United Kingdom, being more closely connected with the operating side than with the manufacturing. The development of aerial topdressing in this country has followed a line more or less unique and is certainly becoming a really significant feature in the economy of the country. In actual fact New Zealand people are extremely air minded. Our internal airways communications are becoming more and more popular. The awkward geographical obstacles, such as the division of the Islands and the mountainous country, are such that air travel brings conveniences and advantages which do not apply to other forms of transport. These advantages have meant that the ordinary member of the population is far more likely to be directly interested in Aviation matters than would be the case in many other countries. Aerial topdressing again has brought directly to the eye of the farmer something of the potentialities of air transport and possibly farmers as a class are far less likely to be enamoured with air transport than almost any other class of person. Since New Zealand is so primarily a farming country it is extremely significant that air transport has in this way come to the attention of the farmer.

The expansion of aeronautical activities in this country at the present time is rapid and shows every sign of continuing to be rapid. It is satisfactory then to find that the Royal Aeronaufical Society is taking an active and in many respects a leading part in this rapid development.

Another interesting item in N.Z. Aviation is the award to Tasman Empire Airways of the Cumberbatch Trophy for safety record. Although neither the Society nor the Division have any direct connection with this Trophy, nevertheless it is indicative of the representation of the Division that T.E.A.L. has many of its staff active members of the Division. These include the T.E.A.L. General Manager as Vice-President, T.E.A.L. Chief Engineer as a past Chairman of Auckland Branch and other members of T.E.A.L. take an active part as office bearers.

In the coming year New Zealand will be definitely on the map with the England to Christchurch Air Race. Although the N.Z. Division is not directly concerned in this, nevertheless again it is indicative of the Division's ubiquity that several members are directly connected with the organisation of the Race at this end.

Quite apart from these direct activities, there is no doubt that the regular meeting of members of the Division one with another is of inestimable benefit. A community of interest and an active understanding of the other person's problems are not the least of these. The opportunity for making and maintaining friendships is also something which the Division supplies of the very greatest value. The active interest of yourself and, in fact of the whole body of the parent Society, is just another aspect of this and this in itself is ample reason enough to send you our very sincere and best wishes for Christmas and the New Year.

Yours faithfully,

T. M. COLERIDGE,

Honorary Secretary,

New Zealand Division of the Royal Aeronautical Society.

Elections

The following is a list of new members and transfers of membership of the Society:—

Associate Fellows

John Alfred Beadon
Terence Beville Adair Boughton
Ronald Claude Graham Boxer (*from Graduate*)
Ralph Brown (*from Associate*)
Eric Leslie Carlisle (*from Associate*)
Eleanor Lettice Curtis (*from Associate*)
Stanley Hargreaves Ellis (*from Graduate*)
Geoffrey Fawn
Alan James Greenhalgh (*from Graduate*)
Stephen John Hancock (*from Graduate*)
Wm. Marshall Hargreaves
Ronald Frederick Hill (*from Associate*)
Edward Henry Holder (*from Associate*)
Thomas George Kent (*from Graduate*)
Bernard Lovegrove (*from Graduate*)
William Muirhead
Peter Jack Palmer (*from Graduate*)
Bertram Anthony Peaster
Theordore Stanley Revell (*from Associate*)
Colin Arthur Richardson (*from Associate*)
Alan Scrimshaw
Harold Warren Shattock (*from Graduate*)
Richard Herbert Taylor (*from Graduate*)
Walter Edward West
Mark Vincent Wignall
Geoffrey Edward Williams (*from Graduate*)

Associates

Cecil Frank Allen
Dennis Arthur Thomas Ball
Gysbert Bloemendaal
Roy Milner Bradley
Leslie Douglas Chapman
James Henry Cruddace
Albert Ernest Dennard
Walter Miller Fitch
Eric Arthur Franks
Neil McLeod Gibson (*from Companion*)
Derek Norman James (*from Companion*)
Bernard John Naish
John Leslie Nunn
James Ronald Plenderleith
Clifford Edwin Refoy
David Royle
Alexander Rodger Sturgeon
Thomas Benjamin Taylor
Geoffrey Thomas White
Arthur Wilby

Graduates

Richard Cameron Aitken
John Chas. Arthur Baldock (*from Student*)
Robin John Balmer
John Robert Burton (*from Student*)
Antony Shui-Lau Chan (*from Student*)
Francis Charles Drinkwater
Ronald Arthur Dutton
Edward Vernon Griffith
Jack Womersley Headley (*from Student*)
Norman Jackson (*from Student*)
Morris Vivian Jenkins
Herbert Judson
John Irvine Lattey
Harold Moore Lofthouse (*from Student*)
William Robert McLean
David Needham (*from Student*)
Denys Randolph
Peter Marcus Ray
Robert Chas. Leslie Sample (*from Student*)
Harold George Sevier (*from Student*)
Harold Alexander Simon (*from Student*)
Vivek Ranjan Sinha
Frederick Thomas Smith
Christopher John Hamshaw Thomas (*from Student*)
Ernest Cyril Thomasson
Denis Edward Weeding (*from Student*)
Rex Maddock Whalley (*from Student*)
Frank Gerrie Willox
Norman James Wilson

Students

Mond Khan Amir
Ronald Frederick Ayers
James Sidney Baker
Martin Charles Bennett
John Arthur Boddy
Thomas Patrick Joseph Walter d'Alton Bourke
Derek William Brown
Derek George Burton
Frank Richard Dugay
George Kasin Ferris
Edgar Ronald Fraser
Ronald Anderson Gellatly
Dennis Gortmans
Geoffrey Leonard Hearne
Reginald Arthur Jackson
Edward Henry James
Brian Greig Jolly
Dennis King
Brian Richard Luesley
John David Francis Mac-Naughton
Afzal Haq Malik
William Colin Frank Moxon
Gordon Oulsnam
Anthony John Pape
Brian William Payne
John Lloyd Hendy Phillips
Frederick Bowen Powell
John Graham Price
John Holdom Stock
Michael Edwin Thomas
William Malcolm Thomas
Dennis Albert Vaughan
Wallace Barrie Dannatt Wardle

Companions

Biaal Chandra Dutta
Paul Leon Albert Raes (*from Student*)
Alan Charles Keith Robertson

Annual Subscriptions

Members are reminded that their annual subscriptions became due on 1st January 1953. The rates are:—

	Home	Abroad
Fellows	5 5 0	4 4 0
Associate Fellows	4 4 0	3 3 0
*Associates	3 3 0	3 3 0
Graduates (aged under 26)	2 2 0	2 2 0
Graduates (aged 26 and over	2 12 6	2 12 6
Students (aged under 21)	1 1 0	1 1 0
Students (aged 21 and over)	1 11 6	1 11 6
Companions	3 3 0	3 3 0
Founder Members	2 2 0	2 2 0

*Any Associate elected before 1st October 1947 may, if he wishes, elect not to receive the Journal, and in this case his subscription will be reduced by £1 1s. 0d. to £2 2s. 0d.

It will avoid delay and confusion if members, when sending remittances for subscriptions, will state their names clearly and give their addresses and grades of membership. Remittances should be made payable to the Royal Aeronautical Society.

Changes of Address

To assist in keeping the records of members correct and up to date the Secretary will be glad if all members will notify him as soon as possible of changes of address.

When notifying changes please give the following particulars:—

Name (in block letters).
Grade of membership.
New address (in block letters).
Old address.

Changes of address must be received *before the 15th of the month* in order to be effective for the Journal for the following month.

Journal Binding

Self-Binder Cases

Self-Binder cases of the "Easibind" type are available from the offices of the Society. These binders are for members who do not have their Journals permanently bound, or who wish to keep their Journals together during the year for binding later.

These cases will hold 12 Journals which are kept in place by means of flexible steel wires. Journals can be inserted or withdrawn easily without damage, so preserving the contents for permanent binding later. The Journals will open flat at any page.

The binder is strongly made in durable dark blue leather cloth on stiff board covers and has gold lettering on the spine. The year is not blocked on the spine but there is a panel on which members who wish to use the binder as a permanent case can put the date.

The cost is 11s. 6d. each including postage and packing for either the size to fit 1952 and previous Journals, or for the size to fit the Journal from January 1953, which has been increased in size. Orders and remittances should be sent direct to the Secretary at the Offices of the Society and it is *important to state whether the old size or new size is required.*

Permanent Binding

There is no increase in the price of permanent binding of Journals. The prices are:—

1952 Volume (including packing and postage 16s. 0d.
Previous Volumes (including packing and postage) 18s. 0d.

Journals, *with a note of the name and address of the sender,* should be sent direct to the Lewes Press, Friars Walk, Lewes, Sussex, and the remittance to the Secretary at the Offices of the Society.

Journal of The Royal Aeronautical Society

WITH WHICH IS INCORPORATED THE INSTITUTION OF AERONAUTICAL ENGINEERS

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The Scope and Limitations of the Photoelastic Method of Stress Analysis

by

H. T. JESSOP, T.D., M.Sc., F.Inst.P.

The 861st Lecture to be given before the Royal Aeronautical Society was held at the Institution of Mechanical Engineers, Storey's Gate, London, S.W.1, on 27th November 1952. Mr. G. R. Edwards, C.B.E., F.R.Ae.S., Vice-President of the Society, presided and introduced the lecturer, Colonel H. T. Jessop, T.D., M.Sc., F.Inst.P., Reader in Photoelasticity at University College, London, who was a pupil of Professor Filon and later worked as Professor Filon's Research Assistant Mr. Edwards said that his company had been fortunate in making use of Colonel Jessop's facilities for some photoelastic analyses of their own pieces of machinery; the photoelastic method of analysis was one which the Aircraft Industry was finding of increasing value.

During the lecture Colonel Jessop demonstrated the stress lines in various shapes of materials with a polariscope.

Introduction

The problem of determining the distribution of stress over the various components of a structure such as an aircraft frame is one which has always presented considerable difficulty. For a large proportion of the components calculations of the stresses can only be carried out on the basis of assumptions and approximations, the effects of which cannot be estimated with any degree of accuracy and, since considerations of size and weight impose limitations upon the employment of a large safety factor, there has always existed a need for some practical means of checking the results of calculations or of obtaining more reliable estimates of the stresses.

One such means is to be found in the photoelastic method of stress-exploration. In comparison with other methods it is rapid and inexpensive, in many cases it gives much fuller and more exact information, and it is moreover the only method by which the magnitudes of the stresses in highly localised stress-concentrations can be determined with any degree of accuracy. This last point is of particular importance in modern aircraft design, for it is becoming increasingly urgent to have a more exact knowledge of the magnitudes of these local stresses in order to avoid the possibility of their leading to fatigue failures. The photoelastic method is also the only one which gives any information as to the stress-distribution at internal points, and while it is usually the boundary stresses which are of chief interest to the engineer, there are cases in which the rate at which the stresses fall off below the surface of the material is likely to have a significant effect upon the possibility of failure.

Outlines of the photoelastic method have appeared in various publications[1], and the general principles of the method are too well known to need recapitulation here, but many developments in materials and techniques have taken place in recent years and these have increased the power and range of the method to an extent which is not yet generally appreciated by the engineers for whose use it has been designed. The new developments have taken place largely in research laboratories which have not the resources to make extensive investigations, but the methods have been applied to enough practical examples to demonstrate their value in a wide variety of stress-problems. In this paper an attempt is made to give an account of the present scope of the method and of the techniques employed, and to indicate some of the types of problem in which it could be used with advantage.

Two-Dimensional Techniques

The two-dimensional method is substantially that developed by Coker and Filon[2] in the years 1912-1930, although the introduction of new materials and new techniques has led to simplified procedure and improved accuracy. This method is strictly applicable only to components whose form is that of a flat plate of uniform small thickness to which the loads are all applied at the edges and all act in the plane of the plate. For such components the photoelastic model is generally examined under load in the ordinary polariscope, and the complete stress-distribution is obtained in most cases rapidly and with considerable accuracy. Measurements of the fringe-pattern seen in the polariscope yield directly (1) the magnitude of the stress at all points on

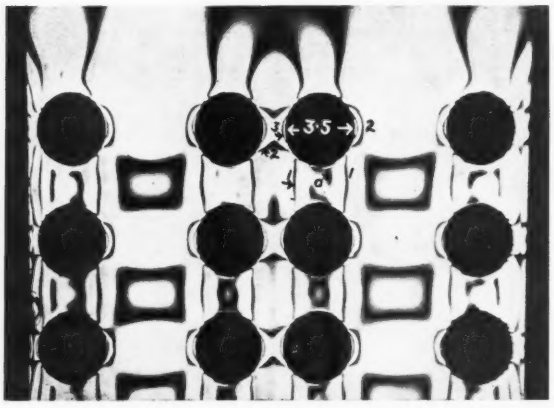

FIGURE 1. Fringe-pattern showing the stress-distribution around an array of holes in a flat plate under tension, and demonstrating that the peak tensions are approximately the same in the wide and narrow ligaments.

the free boundary, and (2) the magnitude and direction of the greatest shear-stress at all internal points. In many practical problems this information is all that is required to check the suitability of a design or to determine the safe load for a component, and often a mere inspection of the fringe-pattern will suffice. If the magnitudes of the separate stresses at any point are required, these may be found by simple calculations based on measurements of the fringe-pattern.

Figure 1 is a photograph of the fringe-pattern in a plane model which was one of those used to investigate the stresses round the steam-pipe holes in a boiler drum. The diameter of the drum was very large compared with the thickness of the shell and a flat plate under uniform tensile load reproduced, with good accuracy, the conditions of hoop tension in the shell. The investigation was undertaken to find the effect of uneven spacing of the holes, and the main interest was in the strains around the holes and the possibility of these being so large in the narrower ligaments as to cause leakage under high pressure. The fringe-patterns in loaded models showed at once that there was little difference between the peak tensions in the wide and narrow ligaments; the fringe-orders at opposite sides of the holes in Fig. 1 are obviously approximately the same. Measurements and calculation of the actual stresses showed that for a wide range of differences in spacing the difference in peak stresses did not exceed 5 per cent.

Figures 2 and 3 illustrate a bolted or riveted cleat joint, a type of problem which might well be investigated by photoelastic methods. In the model shown, the cleat is made of Perspex, the photoelastic sensitivity of which is very small, so that the pattern observed through the whole model is sensibly that due to the stresses in the middle bar. This gives a qualitative comparison of the stresses in such a bar with the two different bolt spacings shown, but even this is sufficient to show that with the closer spacing the bolt loads are more equally distributed, while the peak shear-stress in the bar is reduced.

A full and accurate investigation of any specified pattern of such a joint could be made by using either a reflection method on loaded models, or the frozen stress method. The stresses in the cleat and the bar could then be completely separated and the effects of varying both spacing and size of bolts explored. This preliminary test suggests that there will be in any particular design an optimum spacing, and that this spacing will be closer than that normally employed in such joints.

The two-dimensional technique may also be employed in many cases in which the stress-distribution is not strictly two-dimensional, such as an I-section spar in which the loading is in the mid-plane of the spar and parallel to the plane of the web. Here the stresses in the web will be sensibly two-dimensional and may be deduced with good accuracy from the fringe-pattern,

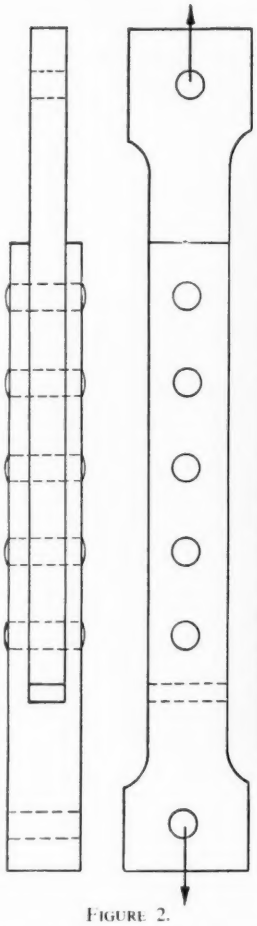

FIGURE 2.

FIGURE 2. Simple model representing a bar attached to a cleat by a row of pins.

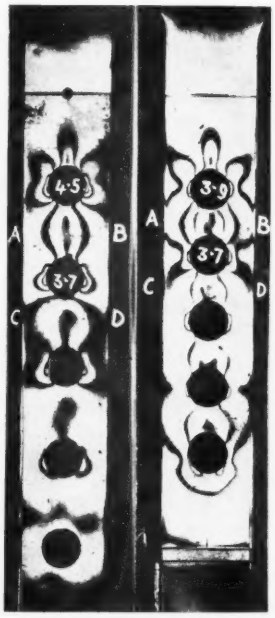

FIGURE 3.

FIGURE 3. Fringe-patterns in the bars of two models similar to that in Fig. 2 under tension. The patterns show on inspection (*a*) that the mean stress across each of the corresponding sections AB, CD is greater in the model with more closely spaced rivets, and therefore that the load is more equally shared by these rivets, and (*b*) that the greatest shear-stress in each bar occurs at the boundary of the first hole, and is less in the model with the closer spacing. The patterns around the lower holes are not significant, for here the higher stresses in the Perspex cleat begin to interfere with the smaller ones in the more optically sensitive bar.

while the pattern in the flanges will allow the estimation of the mean stresses through the thickness of the flange. The stresses in the fillets between flange and web would need a full three-dimensional exploration. In tests made recently on such a spar model by Vickers-Armstrongs Ltd., the photoelastic investigation revealed high stresses which their calculations had failed to indicate, and suggested the type of modification which should be made in the design. A second test, on a model built to the modified design, showed a satisfactory reduction in these stresses. In this case photoelastic tests occupying three weeks yielded better information than was obtained from calculations which had taken a much longer time.

Three-Dimensional Techniques

THE FROZEN STRESS METHOD

The general characteristics of the frozen-stress effect are well known to all interested in practical stress analysis. The features of the effect which are relied upon in stress investigations are first, that the "frozen" photoelastic fringe-pattern shall correspond with the stress distribution in the model under direct load, and second, that no change in the "frozen" stress shall be produced by cutting the model. That these two conditions are, in fact, fulfilled in the two-dimensional case and in simple three-dimensional cases, has been demonstrated on a variety of models. The photographs of Figs. 4 and 5 illustrate these two features of the "frozen" stress. The method renders it possible to freeze the stresses into a model of any shape and, by cutting thin slices and examining them photoelastically, to explore the stresses at all points of the model.

Three main methods of examination are available:—

(1) A thin slice of the model may be examined in the polariscope using the ordinary two-dimensional technique, with the light passing through the slice either normally or obliquely (Figs. 6 and 7).

(2) A narrow pencil of light may be passed through the uncut model (or any part of it) and the stresses at points on the path of the pencil explored by measurement of the fringes which are observed by scattered light[3, 4] (Fig. 8).

(Since this method does not involve slicing the model it may equally well be employed on a model under load, and in certain cases it will give all the information required without necessitating the freezing of the model.)

(3) A thin slice of the model may be examined in polarised light on a universal tilting stage[5]. By making a systematic series of small displacements about two

FIGURE 8. Fringes seen by scattered light in a beam passing along the line A B of the slice from the grooved cylinder shown in Fig. 6. The spacing of the fringes gives a measure of the difference between the axial and circumferential stresses.

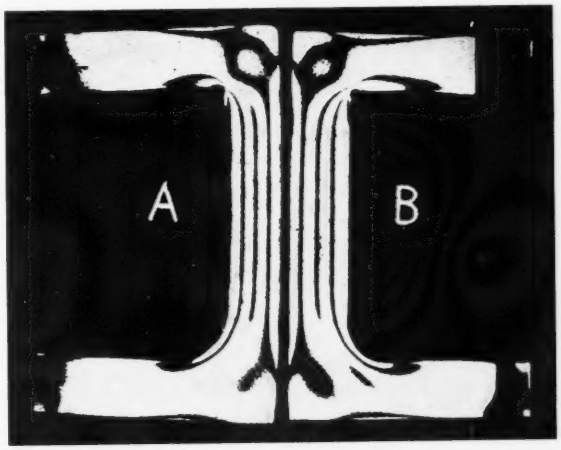

FIGURE 4. Fringe-patterns in two similar models, one (B) under direct load, the other (A) having the stresses frozen into it under a similar load. The stresses were frozen into model A eighteen months before this photograph was taken.

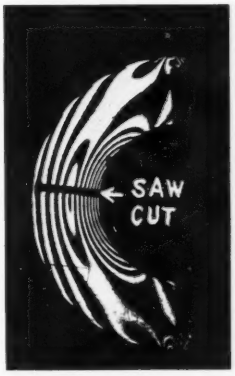

FIGURE 5.

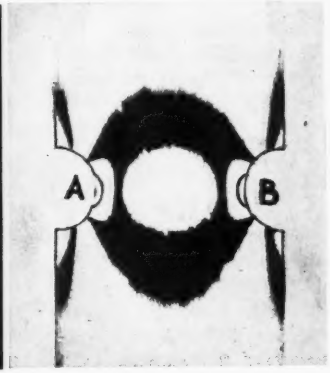

FIGURE 6.

FIGURE 5. Frozen fringe-pattern in a model, showing that the pattern is not disturbed by cutting the model.

FIGURE 6. Fringe-pattern in a slice cut in a diametral plane from a grooved cylinder in axial compression. The fringes give the difference of the principal stresses in the plane of the slice.

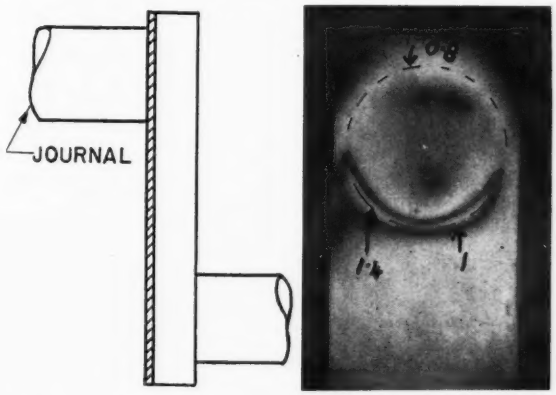

FIGURE 7. Fringe-pattern in a slice cut from the web of a crankshaft. The shaded portion in the diagram indicates the slice. The fringe-pattern shows clearly the distribution of shear stress around the journal.

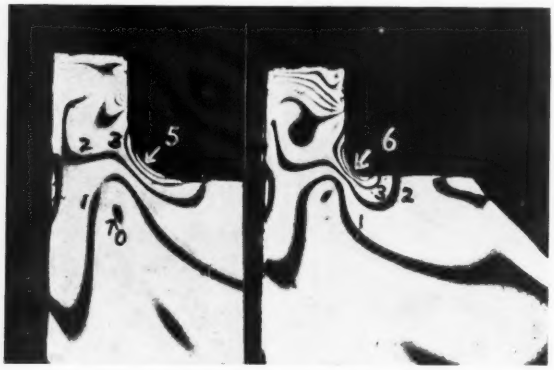

FIGURE 9. Fringe-patterns showing the stress concentrations in fillets of different radii. In this example doubling the fillet radius has reduced the peak stress from 6 units to 5.

perpendicular axes the directions in space of the three principal axes at any chosen point of the slice may be determined.

In most cases the quickest and most accurate approach will be by a combination of two, or all, of these methods of examination and will lead to the determination of the principal stress differences. As in the two-dimensional case the separation of the stresses will, in general, require an integration process.

The time and labour involved in a three-dimensional investigation vary enormously with the type of problem. A complete exploration of the stresses at all points of even a simple model would be a very long process, probably involving the freezing and slicing of two or three models, but for practical purposes the engineer is interested mainly in the stresses in certain specified regions, and in most cases these happen to be the regions in which the solution is comparatively simple. In order of simplicity regions may be classified as:—

(*a*) *Free boundaries in a plane of symmetry.* Here only the magnitudes of two stresses have to be found, and this may be done most easily by a combination of methods 1 and 2, or in some cases by two observations by method 2 without slicing the model.

(*b*) *Free boundary not in a plane of symmetry.* This involves finding the directions of the two stresses, as well as their magnitudes, but the solution is very little more complex than for case (*a*).

(*c*) *Interior regions in a plane of symmetry.* Here the direction of one stress is known. The directions of the stresses in the plane of symmetry and the magnitude of the stress difference are obtained by method 1. The second stress difference is obtained by combining observations by method 2 with those of method 1. A comparatively simple integration leads to the values of the separate stresses[6].

(*d*) *Interior regions not in a plane of symmetry.* In this case the directions of the stresses at any point have first to be found by method 3. The slice used for this purpose will probably be found to have been cut in a plane which is not suitable for accurate measurement of both stress differences, and this will necessitate the use of a second model. Another slice may be cut from this, and the stress differences found. The separation of the stresses in this case involves the integration of measurements taken in two perpendicular planes, and, in general, will involve the use of two more models[7].

It may be noted that in cases (*c*) and (*d*) the most troublesome part of the solution is in the separation of the stresses. In problems where a knowledge of the principal shear stresses at a point is all that is required, these cases present little more difficulty than do the previous ones.

There are available, therefore, the means of measuring the stresses at practically any point of a "frozen" model. The conditions necessary in order to apply these means to the determination of the stresses in an engineering component are, in general, three:—

(*i*) It must be possible to construct a scale model of the component in a homogeneous photoelastic material.

(*ii*) It must be possible to reproduce the loading conditions of the prototype in an annealing oven and to keep these conditions constant during the "freezing" period (16 hours or more).

(*iii*) It must be possible to cut from the model slices sufficiently thin for the stresses in them not to vary appreciably through their thickness.

These conditions being satisfied, the solution to the problem is possible and its difficulty depends only on the complexity of the model and on the amount of information which is required.

Stresses in Fillets

An important and frequently occurring problem in which the photoelastic method gives the required information very easily is that of the stress concentrations in fillets. The available data regarding stress concentration factors in fillets are all based on experiments or calculations in which the fillet is an isolated "stress-raiser" in a test-bar in which the stress distribution in the absence of the fillet is known. In practice, the critical cases generally occur in regions near to points of load application where no basic stress is known to which a concentration factor can be applied

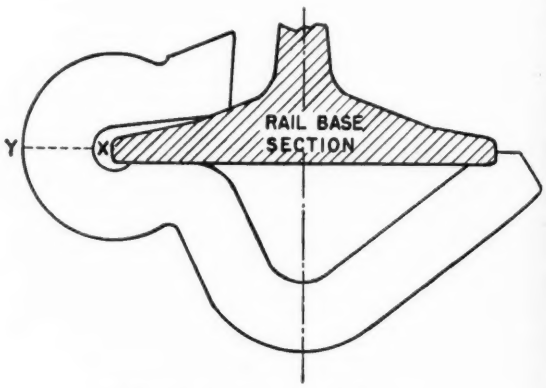

FIGURE 10. Diagram of rail anchor.

An illustration of such a problem was given in a recent paper in *The Aeronautical Quarterly*[8] in which a photoelastic investigation revealed a very high stress concentration in the sharp corner of a spot-facing on an elevator hinge-bracket. In most such components the sharp corners are replaced by fillets, and this will undoubtedly lead to a reduction in the peak stresses, but the exact amount of the reduction is still unknown and is likely to vary widely with the design and loading of the particular component. The choice of a radius for the fillet, therefore, is a matter of guess work backed by experience, and the designer is still in ignorance of the actual peak stresses involved.

Figure 9 shows a comparison of the stresses in fillets of different radii in a loaded bracket. Two-dimensional models have been used in this illustration in order to obtain fringe-patterns from which the stresses may be compared by inspection, but equally good information may be obtained for three-dimensional models by taking accurate measurements of the smaller optical effects which appear in thin slices cut from the models.

Transference of Results from Model to Prototype

One of the first questions raised by the engineer in relation to photoelasticity is that of the effect of the material of which the model is made upon the stress distribution. In this connection the common description of photoelastic materials as " plastics " tends to be rather misleading. In fact the stress-strain relation in these materials is strictly linear for a wide range of loads, extending in most materials right up to the point of fracture, and the fractures, when they occur, are brittle ones. This perfectly elastic behaviour has been confirmed in many two-dimensional cases by a comparison of the stress distribution obtained from the photoelastic pattern with that given by the exact theoretical solution for a perfectly elastic material. The only respect in which two-dimensional photoelastic materials show any plastic behaviour is in their being subject to a certain amount of creep with time, both in strain and in the stress-optical effect. In the materials chosen for two-dimensional stress investigations this creep is small and the greater part of it takes place during a comparatively short period after applying the loads. In most materials, also, the magnitude of the creep is proportional to the stress and so there is no change produced in the relative optical effects at different points of the model. The only effect of the creep, therefore, is to produce a small change in the stress-optical coefficient of the material, and this may be allowed for by delaying measurements on both model and calibrating test-piece until the rate of creep has become negligibly small. The frozen-stress materials exhibit no creep at their critical " freezing " temperature, and the recovery creep after removing the loads is negligible.

The stress distribution given by photoelastic methods therefore in both two- and three-dimensional investigations, is that which would occur in the prototype if the material of which the latter is made retained a linear stress-strain relation throughout the range of the stresses operating. If the peak stresses, as indicated by the photoelastic method, are higher than the yield stress of the prototye material, then the latter will undergo plastic yield in the most highly stressed regions with a consequent redistribution of stress which can only be estimated. The magnitude and extent of the high stresses in the elastic distribution, however, give some measure of the amount of plastic yield which will occur in the prototype.

FIGURE 11. Fringe-pattern in curved part of anchor.

The rail anchor illustrated in Fig. 10 provides a good example of such a case. The anchor, as indicated in the diagram, clips on to the base of the flat-bottomed rail where it is held by the spring grip of its jaws. The face of the lower part of the anchor bears against a sleeper, preventing longitudinal creep of the rail. It was found necessary to modify the shape of the anchor and a photoelastic test was undertaken to determine the type of modification which would produce the best " grip " on the rail. Fig. 11 is a photograph of the fringe-pattern in the curved portion of one of the models examined. On scaling up the stresses observed in the model it was found that the peak stress in the anchor, on the basis of an elastic distribution, would be very much greater than the yield-stress of the steel used. The distribution across the critical section XY of the anchor would therefore be something of the type indicated by the continuous graph in Fig. 12, in which only the stresses in the portion EF of the section would contribute to the grip.

The crucial factor, therefore, was the amount of plastic yield which would occur and, since the criterion of yield might be taken to be the magnitude of the shear stress, it was not necessary to carry out any separation of the stresses, but only to compare the shear stress distributions in different models. Fig. 13 shows the graphs of the shear-stress across the critical section XY for three different models tested.

EFFECT OF POISSON'S RATIO

One other factor may affect the transference of results to the prototype and that is the value of Poisson's

ratio for the materials. In two-dimensional problems the stress distribution is a function of this elastic constant only in the case in which an unbalanced force acts on the boundary of a closed hole in the model. Even in this case the effect is not large, for the value of Poisson's ratio for the materials used in two-dimensional work does not differ much from that for most structural materials. Results in typical cases for which the stresses can be calculated indicate that errors from this cause are not likely to exceed 7 per cent[9].

The value of Poisson's ratio under frozen stress conditions, however, is very nearly 0.5, and in general this must have some effect upon the distribution of the stresses. Exactly how large this effect will be in any particular case it has been impossible, up to now, to estimate, for there are no theoretical solutions to compare with practical observations. The development of the scattered light method of observation does now, however, offer a means of assessing this effect, for it will now be possible to use this method on a directly loaded model made of the cold loading type of material and compare the results with those obtained from a similar frozen model. A cold loading material with the requisite light-scattering properties is now being produced, and experiments will be started shortly which, it is hoped, will result in a reduction in the possible errors due to this factor.

The accuracy of the Photoelastic Method

The accuracy obtainable in a photoelastic investigation necessarily varies enormously with the nature of the problem, and it is impossible to lay down any general limits. All that can be done is to examine the sources of error and to estimate in each individual problem the possible effect of each on the determination of the stresses in which we are interested.

The accuracy of the actual optical observations is generally extremely high. Measurements by direct visual observation may be made to an accuracy of better than 0.01 of a "fringe" and the use of photometric methods reduces this possible error to something of the order of 0.002 fringe[10]. The peak stresses even in a thin slice from a frozen model, will generally produce an optical effect greater than half a fringe, and consequently errors from this source may be considered negligible. In the case of a thin slice, for example, the error in measurement of the thickness of the slice is likely to be greater than that in the optical observations.

The main sources of error appear in the photoelastic material itself, in the construction of the model, and in the reproduction of the loading conditions which apply in the prototype. The material may be subject to lack of homogeneity and to initial stresses which are bound into it by chemical action during its manufacture. These defects can only be avoided by the exercise of great care in the casting[11]. The models used for three-dimensional work are either machined from a solid block of material, or cast in a mould. The heat generated during polymerisation of the resins employed makes the casting of large blocks which are homogeneous and free from initial stresses a matter of some difficulty. The casting of a

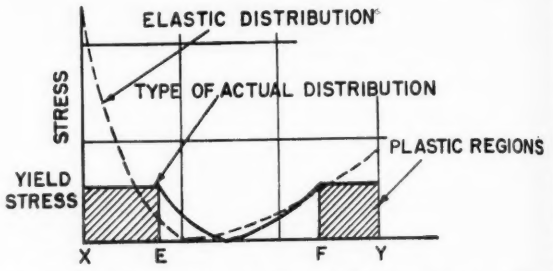

FIGURE 12. Comparison of stress distribution in the photoelastic model of the rail anchor and in the anchor itself.

model in or near to its finished shape, by reducing the mass of reacting material, helps to minimise the temperature effects. The technique of casting models in this way is still in the experimental stage, but enough progress has been made to show that in many cases it is quicker and more satisfactory than direct machining.

Figure 14 shows a photograph of a model of a "tucked-in" porcelain insulator cap, one quarter full size, which was cast directly in a mould. The investigation was to determine the peak tensions in the cap which would be produced by the mechanical forces needed to hold it in position and produce an air-tight joint. The model was frozen under load, and measurements on a slice cut in a plane of symmetry yielded the necessary information.

It may be mentioned here that one of the most urgent needs in photoelasticity today is for some systematic work by skilled "plastics" chemists on the development of photoelastic materials. The properties required for this application are different from those required for industrial uses, and although the firms manufacturing the resins have given much useful information and help, most of the research up to the present time has been done by physicists and engineers having very little knowledge of the chemistry of the subject.

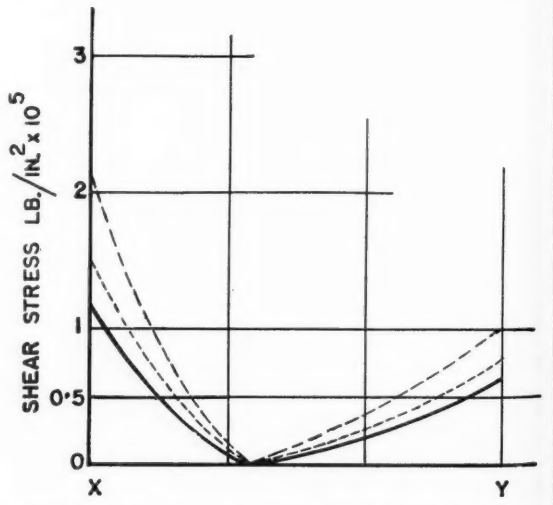

FIGURE 13. Graphs showing stresses across section XY of three different models of anchor.

In both directly loaded two-dimensional models and in slices cut from a frozen model it is possible to apply corrections for initial stresses, but since such stresses may indicate a lack of homogeneity in both elastic and photoelastic properties of the material, these corrections cannot be relied upon unless the initial stresses are small. It is accordingly frequently necessary to discard models and blocks of material, and this still leads to a great wastage of time and material. If, however, care is taken in the selection of the materials, experiments have shown that excellent agreement is obtained between measurements taken on several similar models.

The reproduction of loading conditions is, in many cases, the most serious source of error. The relatively large strains which occur in the model, especially in frozen stress conditions, may in some cases tend to produce serious differences in the distribution of applied loads. No general rules for avoiding such errors can be laid down. The conditions of each individual problem must be studied and some means devised of applying the loads in such a way as to minimise the differences in loading. In some cases, for example, very small loads must be used for investigation of stresses in the vicinity of load points and larger loads for the more remote regions. In others it may be possible, by studying the effect of varying the loading on the model, to obtain a good estimate of the stress distribution under the conditions existing in the prototype.

As an estimate of the order of accuracy to be expected from photoelastic investigations, one might say that the peak stresses in a straight forward two-dimensional model would be liable to an error of from one to 5 per cent., rising to a possible 10 or 12 per cent. in cases where unbalanced forces act on the boundaries of holes, and to an error of from 5 to 30 per cent. in a three-dimensional model. While the possible error in the last case may appear to be high, it must be borne in mind that these highest possible errors will occur chiefly in problems for which no other method offers a solution of any kind, and that in such cases even a qualitative comparison of the stresses in two different designs may be of considerable value.

Future Developments

There is no doubt that the methods and techniques outlined here will undergo great improvements as more workers enter the field and apply them to a wider variety of problems. In this country developments have so far taken place largely in research laboratories, and engineering firms, on the whole, have been slow in taking up new methods, and not a little sceptical about such reports and results as have come to their notice. Thus, while it may fairly be said that, during the past few years, so far as fundamental research is concerned, the leeway lost to America in this subject during the war years has been made up, we still lag a long way behind in the matter of practical applications.

There are signs, however, that the value of the photoelastic method is becoming more widely appreciated and the Stress-Analysis Group of the Institute of Physics, which has been largely responsible for the co-

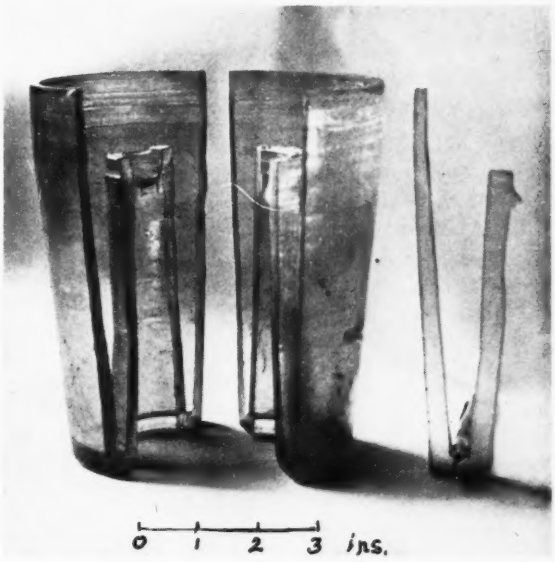

FIGURE 14. Model of insulator cap cast in Marco Resin, showing part of slice cut in a plane of symmetry.

ordination of work in this field, now has members representing a good proportion of the large engineering concerns in this country.

As regards future development, there is one technique which is being investigated at the College of Aeronautics which offers a possible approach to the use of photoelastic methods in measuring the strains in actual components. In this method a layer of photoelastic material is cast on to the surface of the component itself, and the optical effect in the layer when the component is loaded is observed by a reflection method. The photoelastic material thus provides a continuous strain-gauge over the surface. Satisfactory tests have been made on a number of test models, but so far as the author is aware, the accuracy and reliability of the method in general investigations have not yet been fully established. Difficulties of observations of the effect in some cases will probably limit the range of application of the method in actual investigations, but it offers the possibility of a means of measuring the amount and extent of plastic flow at the surface of a component, which might usefully supplement other methods of investigating the behaviour of the actual structural materials.

Conclusion

It is not suggested that photoelasticity will provide the solution to all stress distribution problems, or that a solution, when possible, will always be a quick and easy one. The aircraft designer especially will appreciate not only the need for using all available methods if he is to cover his wide variety of problems, but also the fact that the solution to many of these will be hard to achieve by any method. This paper has endeavoured to show, however, that there are certain vital problems in which photoelasticity will yield reliable information which no other method can give,

and other problems in which the photoelastic method will be quicker and easier than any other known at present.

ACKNOWLEDGMENTS

The author wishes to acknowledge his indebtedness for permission to make use of the undermentioned examples as illustrations in this paper:—

The Boiler Drum—Babcock & Wilcox Ltd.

The I-section Spar—Vickers - Armstrongs Ltd., Weybridge.

The Crank Shaft Web—Mr. D. S. Rabley, Devonport Technical College.

The Rail Anchor—The P. & M. Company (England) Ltd.

The Insulator Cap—Micanite & Insulators Ltd.

Thanks are also due to the technicians of the photoelastic laboratory and to members of the staff of the Civil and Municipal Engineering Department at University College, London, for much of the practical work which was involved in the preparation of the paper.

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Discussion

R. L. LICKLEY (Fairey Aviation Co. Ltd., *Fellow*): Some of the photographs shown must have seemed like magic to people who had learned stressing before there was a photoelastic method. The Society was to be congratulated on having asked Colonel Jessop to lecture on that most important method, which was one which should be used to a greater degree in any work connected with stress and strain distribution.

Some people doubted the effectiveness of the method, and he was very glad that a good portion of the paper had been devoted to answering the sort of questions which might be asked. The usefulness of the method depended on how the answers obtained from a piece of photelastic material could be related to the actual material which would be used in a structure; he felt that they could now say that those answers were fairly closely in agreement. In many cases to use the words "fairly closely" was to be conservative; it might almost be said the answers were exactly in agreement.

Two matters in connection with the method which might be of great value to the aeronautical engineer were weight saving and fatigue loading.

There were many occasions on which the engineer was unwilling to reduce the size or section of various parts of an aircraft because he was unable to calculate the stress to any degree of accuracy, and he could afford neither the time nor the expense of testing the parts; further, a part might not be available for testing for quite a long time. The use of one of the many types of photoelastic investigation which Colonel Jessop had demonstrated, however, would enable the engineer to find the places in a piece of material where the stresses were too low and also, where they were probably too high. The value of the method in helping to save weight was potentially great.

There was no easy way to obtain a complete answer to fatigue loading and fatigue stressing, but they did know that in many cases fatigue failures arose from bad detail design, which permitted high stress concentration. Colonel Jessop's illustrations, particularly those of fillets and holes, showed how the method indicated where the high stresses occurred; and alternatives could be found quickly. On those two grounds alone the aeronautical engineer should use the method to a far greater degree than he did.

The method had much value in teaching. When he was a student he had found it difficult, and still did, to visualise strain and stress lines; it might not be easier to see them in colours, but it seemed easier. He thought that the universities might well devote more attention to the method and its use as a teaching medium.

Could Colonel Jessop include, as an appendix to the paper, some information on materials? One of the biggest problems in extending photoelastic work in the United Kingdom was probably the difficulty of the supply of useful materials which were reasonably inexpensive and at the same time easy to get. Two years ago, when he was last closely interested in that field, materials were neither easy to get nor were they

cheap. Therefore, if some data on that matter could be included in the paper it would be still more valuable.

Colonel Jessop had mentioned the work at the College of Aeronautics with reflected light, which made possible the measurement of strains on the surfaces. Some of the people connected with that work were present at the meeting; he hoped that they would take part in the discussion, because that was another useful approach to the problem.

DR. R. B. HEYWOOD (Royal Aircraft Establishment): The potential advantage of photoelasticity had been demonstrated in that a wide range of stress problems could be solved by employing modern techniques. In effect the impressive claim had been made that all three principal stresses and their three directions could be found at each and every point in complicated three-dimensional models. That information was more than the designer was prepared to digest, so that in effect the scope of photoelasticity had far outstripped the requirements of the designer.

From the designer's point of view, the impression might be gained from a casual glance at the paper that techniques were too complicated to be of practical use. This was in contrast to the simplicity of the intrinsic problem which the designer wished to solve. Basically, he wished to know the stress at the point which was most likely to fail. Interest in stresses at other points was only secondary.

Practical experience showed that failure almost invariably started from the boundary of a part, rarely from points within. The major problem was thus the determination of the stresses on external and internal boundaries. The solution was obtained directly from the fringe pattern, since the boundary stress was proportional to the fringe order. Thus no greater skill or complexity was involved than merely counting fringe orders. Even with three-dimensional frozen stress models, the boundary stresses could still be found with little further elaboration. Colonel Jessop's method (1) in the Section "Three-Dimensional Techniques" was sufficient, there being no necessity to use scattered light as given in method (2). Admittedly there were a few cases in which stresses were required on a boundary that was in contact with an adjacent part, and then a more advanced analysis became necessary. However, it was significant of the type of problem usually encountered that all except one of Colonel Jessop's examples had their strength determined uniquely by the maximum stress at a free boundary and not at a loaded boundary, nor even at a point below the boundary. Thus the greatest contribution that photelasticity could give was by the use of simple techniques.

The simplicity of technique could be matched with simple apparatus. In Coker and Filon's classical researches on photoelasticity it was necessary to use a complicated polariscope, employing several lenses in a system lined up on an optical bench. Today the lens type of polariscope was still the best type to use for research investigations in which a complete and accurate stress analysis was required, particularly if the polariscope were used in conjunction with Brown and Hickson's photometric method of measuring light intensities. However for a straightforward application of results to design, the newer techniques had made it possible to dispense with lenses by using a ground glass screen for diffusing light. There were several advantages from the so-called diffuse light polariscope, such as no loss of time in setting up and aligning lenses, better rendering of fringes in direct vision, equal accuracy in counting the fringes, and compactness and cheapness. The principal objection to this polariscope was caused by convergency of light, but this condition did not arise when considering individual points as distinct from areas of the model. Both the scope and application of photoelasticity could be increased by this simplification of technique and apparatus.

The photoelastic method, like all methods of stress analysis, could not be used to indicate the static strength of structures. If structural loads were excessive, plastic flow would occur which would modify stress distributions. Thus the true elastic stress distribution was not applicable. That was confirmed by Colonel Jessop's implication in the results shown in Fig. 12 that maximum stresses of whatever value were reduced to a common yield stress. He thought the author would agree that that was actually an over-simplification, since a part possessing a severe stress concentration was usually, although not always, weakest in static strength. The photoelastic solution was at best of qualitative value only in its application to structures.

In certain other directions, such as in the determination of the first point and extent of yielding, and in the confirmation of theories and mathematical solutions of stress concentrations, photoelasticity could be of great value.

The method could be applied with much greater precision to fatigue problems in aircraft, for the fluctuating stresses due to gusts, and so on, were fully affected by stress concentrations when failure occurred in a large number of cycles. That important application had been emphasised by Colonel Jessup.

One potentiality of the method which might have received more attention in the paper, was that photoelasticity should be used not only for the assessment of stresses in a given design, but for the improvement of designs. A knowledge of stress distributions enabled certain new techniques and procedures to be employed, whereby perfection in shape could be achieved by ensuring that the stress distribution conformed to a given idealised pattern. The example given in Fig. 9 was not typical in that respect; the modest reduction in stress from 6 to 5 units obtained by increasing the fillet radius could have been further reduced by other refinements in design. The photoelastic method could be used as a standard design procedure for evolving designs of maximum strength, in addition to its more common application in the measurement of stresses.

CAPTAIN J. LAURENCE PRITCHARD (*Honorary Fellow*): This was an unsatisfying lecture to those to whom photoelasticity was not well-known and to those to whom it was. In the latter case it was preaching to the

converted, which was of little use to the former who would not know what the sermon was really about. The lecturer complained, "In this country developments have so far taken place largely in research laboratories, and engineering firms, on the whole, have been slow in taking up new methods, and not a little sceptical about such reports and results as have come to their notice."

What could the lecturer himself expect when his own lecture was so definitely-indefinite ?

"In comparison with other methods," he said in his second paragraph, "it is rapid and inexpensive, in many cases it gives much fuller and more exact information, and it is moreover the only method by which the magnitudes of the stresses in highly localised stress concentrations can be determined with any degree of accuracy. This last point is of particular importance in modern aircraft design, for it is becoming increasingly urgent to have a more exact knowledge of the magnitude of these local stresses in order to avoid the possibility of their leading to fatigue failures."

That was definite enough, but when the accuracy of the photoelastic method was discussed later, the author hedged. ". . . accuracy . . . varies enormously with the nature of the problem . . . The main sources of error appear in the photoelastic material itself . . . No general rules for avoiding such error can be laid down . . ."

The lecturer knew his job well, but sitting on the fence would not help those who read the lecture in Australia, Canada and the Outer Isles and would like to know more. It would be interesting to know, for example, what material or materials were used ? Odd that a lecture dealing with the peculiarities of certain materials under stress did not give, specifically, even one material !

The main credit for the first practical developments for the determination of stress by optical methods must lie with E. G. Coker and L. N. C. Filon, following the earlier investigation of Clerk Maxwell.

On 20th November 1918 Filon took the Chair at a lecture before the Society by Major A. R. Low on the work of the Chairman and Coker, who was in the audience and took part in the subsequent discussion. Coker himself, exactly a year previously, had lectured on Photoelasticity for Engineers at the Institution of the Automobile Engineers.

At the end of his lecture, Major Low said his lecture should be regarded from two points of view. "In the first place as a method already, within limits, in full working order and immediately available as an instrument for obtaining numerical values in particular engineering applications, and as such of immediate commercial value.

"In the second place, it is to be regarded as a method for whose further and fuller development research should be encouraged and subsidised."

Was it ? One only knew that on the death of Filon and Coker the research work in the subject at University College appeared to die as well, and that between the wars the chief research papers were published in America, Germany and France.

A Scotsman, Sir David Brewster, who did so much to found the British Association, was the first, in 1816, to draw attention that glass and similar isotropic transparent solids, exhibited the characteristic colours of crystalline plates under stress. Another Scotsman, Clerk Maxwell, made the first attempt to explore, by this method, the stress system in the plane of a strained plate; and Professors Carus Wilson, Coker and Filon carried out the further experimental investigation described in Major Low's historic lecture before the Society just over thirty-four years ago.

The liaison between research and practical achievement in photoelastic methods was somewhat slower than it had been in atomic weapons. The gaps between research work at the Universities and the application of results in industry could be narrowed very appreciably by both sides doing a little research on each other's propaganda methods. The lecturer had the opportunity in the JOURNAL of the Society to put the subject across in a big way to aeronautical engineers. One hoped he would do what he undoubtedly could do in this direction, with the full authority of his own work on the subject and the knowledge of being one of Filon's pupils.

V. M. HICKSON (Royal Aircraft Establishment): He did not agree that photoelasticity was the only method by which the magnitudes of the stress in highly localised stress concentrations could be determined with any degree of accuracy, especially in view of the subsequent statement that the peak stresses might be subject to an error of from 5 to 30 per cent. It was their experience that the claimed accuracy of brittle lacquer methods of 15 per cent. held under most conditions, and they offered an alternative unless the surface radii were extremely small. Small radii were naturally avoided wherever possible in good design. There were many cases where the stress concentrations were inaccessible, or where contact stresses were involved, in which photoelasticity was the only possible method.

He also differed with the statement that the magnitudes of the two stresses at the surface in the case of free boundaries in a plane of symmetry were best determined by a combination of observations on a slice, and the scattered light method developed by Weller and considerably improved by Colonel Jessop. The oblique incidence method of Drucker, which he had described in two papers*, could be used with good accuracy in the case of unknown surface principal stress directions, and hence the method was even better adapted to the case quoted. It seemed to him that the combination of a slicing method with a scattering method was not so convenient as the slicing method alone, which offered a means of exploring the surface stress in the vicinity of a stress concentration and thus covered cases (*a*), (*b*) and

*Photoelastic Determination of Free Boundary Stresses on Frozen Stress Models by an Oblique Incidence Method. *Brit. J. App. Phys.*, Vol. 2, pp. 261-269, September 1951, and Errors in Stress Determination at the Free Boundaries of Photoelastic Models, *Brit. J. App. Phys.*, Vol. 3, pp. 176-181, June 1952.

(*c*). If the rate of change of stress in an inward direction from the stress concentration were required, the method still applied. It did not apply to case (*d*), which was the determination of stresses of arbitrary direction in the interior, since a knowledge of one principal stress direction was necessary. He would suggest that in the majority of cases it was not necessary to know the interior stresses, since, in general, mean stress might be found from the applied loads and the conditions of equilibrium.

It seemed to him that the proposal for using the scattered light method on cold models with a view to reducing the effect of σ on the results had distinct possibilities. He thought that the real virtue of scattered light was in this aspect. The non-correspondence of σ in the model and prototype, coupled with large deformations, was a perennial problem to workers in the field. There were some theoretical solutions in which the effect of σ could be found—he was thinking in particular of solutions in Timoshenko and of Neuber's notches—and it appeared that the effect of σ was greatest on the lesser of the two principal stresses at the surface. Those cases had been examined and he had three propositions for consideration:

(*i*) The percentage effect of change of σ was most significant in principal stresses on planes at right angles to the principal stress trajectories originating at the loaded boundary.

(*ii*) The maximum effect at a point occurred in one or other of the lesser principal stresses.

(*iii*) For practical purposes, the dependence of the stress on σ at any point was linear.

He emphasised that those propositions were based on known cases and could not have a theoretical proof.

The conclusions were that the maximum stress was not affected to the greatest extent, and that extrapolation from results on two models of different σ should be feasible.

He agreed with Colonel Jessop's remarks on the need for some research into photoelastic materials and would like to put forward a suggestion about frozen stress materials. The effective σ for those materials was about 0·5, which might be attributed to the reduction of Young's modulus as the temperature was raised, without a corresponding reduction in bulk modulus. Dr. Barwell* had given a theoretical expression for σ in a porous material, regarding it as a highly redundant structure. In the limit the value was $\sigma/(2+\sigma)$ *i.e.* 0·2 if $\sigma = 0{\cdot}5$, which was verified approximately by tests on foamed materials†. If a porous plastic could be made with ratio of void-volume to total volume about a half, σ would be reduced to about 0·3. It would be necessary for the voids to be of sub-microscopic size to avoid scattering of light. Alternatively, if they were of microscopic size, the scattering would be useful in the scattered light method.

In the paper attention was drawn to the back reflection method applied to actual components. He believed that the work of the College of Aeronautics was on plane specimens and the investigation was on yielding and plastic flow with consequent high values of strain. He was doubtful whether observations at boundaries on plane stress models were not subject to high error, since the necessarily thick photoelastic material was not in the same strain condition through the thickness. Moreover, if the method were applied to three-dimensional models, the main points of interest were stress concentrations with small concave radii, where conditions were extremely unfavourable for uniformity of stress through the thickness. In his opinion problems of that type were best covered by the brittle lacquer technique: its field of usefulness in the elastic range was exactly that covered by the surface layer technique, and its accuracy was probably higher.

Colonel Jessop had done great service to the Aircraft Industry by drawing attention to the available photoelastic methods which would be of use to the Industry, and also in endeavouring to assess the limitations. It was clear that the three methods of measuring strain and stress, strain gauges, photoelasticity and brittle lacquers, all had their applications to aircraft, and an organisation concerned with aircraft would not be said to be well equipped unless it could cope adequately with all three.

F. A. KERRY (Saunders-Roe Ltd., *Associate Fellow*): In connection with the diagram showing the use of the photoelastic method for determining the distribution of stress around a series of holes in a cleat specimen, there would be a different distribution if the two parts of the joint were of different materials, say one part of light alloy and the other of steel. Was it possible to use photoelastic materials with different characteristics to represent this effect? He supposed it might be done by making the thickness of the portion representing the steel about three times the correct thickness, but the effectiveness of this might be spoiled by the Poisson's ratio effect.

G. R. EDWARDS (the Chairman): He was at a disadvantage in relation to one or two comments which had been made about stressing aeroplanes in certain parts of the structures. All he could do was to support what Colonel Jessop had said, that what he did was quicker and cheaper and more accurate than the method used by Vickers-Armstrongs Ltd.

On fatigue, the big bogey was the unloaded hole and other disturbances in the basic section of the spar, and he was quite confident that any device which could indicate in the very early design stages the sort of stress raisers which were being built into an aeroplane would make a big contribution, because they had not yet heard anything like the last about those problems, but only the beginnings of them.

S. SCOTT HALL (*Fellow*): The pace of advance being what it was in aviation, there was a great risk of the Royal Aeronautical Society spending so much time considering those advances and the applications of engineering to the art of flying that it did not give sufficient attention to fundamental advances in engineering

*R. & M. 2467, 1951.
†R. & M. 2686, 1952 (Pullen).

science itself and to the tools used to design and build aircraft; it was extremely important that now and again a lecturer such as Colonel Jessop should remind them of the advances in those fundamental techniques. The Society was indebted to Colonel Jessop, not only for his lecture, but also because he had instituted courses at University College on the subject, of which some present had taken advantage, and no doubt many more would do so in the future.

Much had been said of Professor Filon and Professor Coker; in 1924 Professor Coker had lectured before the Institution of Aeronautical Engineers, one of the bodies now incorporated in the Royal Aeronautical Society. On that occasion he had said that the science and art of engineering design had been epitomised by Oliver Wendell Holmes, in "The Deacon's One Hoss Shay," from which he had quoted, pointing out that the principles on which it was constructed were a counsel of perfection:—

"'Fur,' said the Deacon, ''tis mighty plain
that the weakes' place must stan' the strain;
Ah! the way t' fix it uz I maintain is only jist
To make that place uz strong uz the rest.'"

That had been borne out by Colonel Jessop's lecture very forcibly.

D. A. THURGOOD (Vickers-Armstrongs Ltd., Weybridge, *Graduate*), *contributed*: Colonel Jessop had rightly stressed the need for research into the development of new photoelastic materials, and it seemed that these developments must take place before the methods and techniques of testing and analysis underwent any great improvements. To the aircraft designer the three dimensional methods offered perhaps the brightest prospects of enlarging his knowledge, or confirming his assumptions of the stress distributions within the structure. So far, however, nearly all three-dimensional work had been on what might roughly be termed "solid" members, illustrated in the paper by the bracket in Fig. 9, and the rail member in Fig. 11. The three-dimensional model visualised by the aircraft designer was a reinforced cylindrical shell representative of wing or fuselage structure from which it was hoped to determine spar boom and stringer end loads, spar web, rib and skin shear stresses in the case of the wing and frame bending stresses, stringer end loads and skin shear streses in the fuselage. In the construction of this type of model the casting technique could be employed, although not without difficulty, and there would appear to be considerable scope for the built-up type of model employing synthetic adhesives. Under test the loading could be arranged to give a reasonable number of fringes in the end load and bending members, but it was more than likely that less than one fringe would develop in the shear carrying members, which, for maintaining correct relative stiffnesses, were necessarily thin and the stress distribution could only be determined by laborious methods of compensation. It would appear that improvements in this direction could be affected by a range of photoelastic materials of similar mechanical properties, but with wide variation in the material fringe values. At present, two materials with different fringe values generally had different mechanical properties also.

In his paper, Colonel Jessop mentioned that photoelastic materials were not strictly "plastic." In view of the increasing application of plastics throughout industry, might there not be a need for a "photo plastic" material?

Colonel Jessop's Replies

In replying rather more fully to the speakers who contributed to the Discussion, he would like first to emphasise that the main purpose of the lecture was to bring to the notice of the Aircraft Industry the great advances which had been made in recent years in the photoelastic method of stress exploration, and to present some idea of the types of problem to which it could be applied and the sort of information a designer could expect from a photoelastic analysis. He had no intention of "preaching to the converted" (even though he might possibly feel that some of these were not quite confirmed in the true faith) and while he very much appreciated the presence of a number of practical photoelasticians, his lecture was not addressed to them. Rather, he wished to convert such unbelievers as there might be among the senior men from the Aircraft Industry—the men in whose province it would be to authorise and support the establishment of a photoelastic laboratory in their respective firms. (He would return to this subject in his reply to Captain Pritchard's comments.) Because of this he thought that no apology was needed for his omission to deal with most of the technical aspects of the subject.

MR. LICKLEY: He was concentrating on and trying to improve Marco resin. Of the other available materials for three-dimensional work, the only satisfactory one in his experience so far was American Fosterite. It was very difficult to obtain that material in large pieces, however, and it did not appear to be materially better except that it had a slightly higher tensile limit than the Marco resin, which could be obtained in this country and which one could cast for oneself. He felt that progress in three-dimensional work, particularly in frozen stress work, was going to be rather on the lines of keeping to low loads, small deformations and more accurate measurements of stresses, than on the lines which the Americans tended to follow, *i.e.* of having bigger and bigger models, heavier and heavier loads and thick slices showing full fringe patterns. He thought that they should stick to loads which conformed more nearly to the loads on the prototype. He recommended Marco resin, but it still needed a lot of investigation.

Written reply: The following was some general information on materials.

(*a*) *For "two-dimensional" investigations*

The three most useful materials they had found

were Columbia Resin (C.R.39), Xylonite and Bakelite (B.T 61/893). For most general purposes C.R.39 was satisfactory. It was obtainable in polished sheets up to 2 ft. × 1 ft., in thicknesses from $\frac{1}{16}$ in. to $\frac{1}{2}$ in. Selected sheets were sensibly stress-free. The cost was roughly proportional to thickness. Sheets $\frac{3}{16}$ in. thick cost £3 per sq. ft. (Obtainable from Ashdowns Ltd., St. Helens, Lancs.)

Xylonite was much less optically sensitive than C.R.39, but was less brittle, was obtainable in very large sheets in a wide range of thicknesses, and was much cheaper. Sheets had to be carefully selected for freedom from stress. Built-up models were easily made by cementing with acetone. The cost varied with thickness. (Obtainable from B.X. Plastics Ltd., Larks Hall Road, London, E.4.)

Bakelite had slightly greater optical sensitivity than C.R.39, and about three times its tensile strength. Obtainable in sheets 11 in. × 7 in. × $\frac{1}{4}$ in. The sheets needed machining and polishing, and also a rather long annealing process to remove initial stresses, and there was a considerable amount of waste in stressed edges. The cost was very high—about £10 10s. 0d. per lb., and an import licence was needed for purchase from America. (Now marketed by Catalin Ltd., Waltham Abbey, Essex.)

(*b*) *For "three-dimensional" work (frozen-stress)*

Here the three most reliable materials they had tried were Bakelite, Fosterite and Marco Resin. The last was the one on which all their work was being done at present.

The Bakelite was the same material as for two-dimensional work, but the sheets could be obtained in thicknesses up to $1\frac{1}{4}$ in. for use in frozen-stress work.

Fosterite was another American material, obtainable from the Westinghouse Corporation, Regent Street, S.W.1. It could be supplied in cylinders up to 8 in. diameter and 2 or 3 ft. long. The rods contained initial casting stresses and required annealing. Annealing and "freezing" must be done in an inert medium (*e.g.* silicone oil). Cost high, approximately £7 10s. 0d. per lb., and an import licence was needed.

Marco Resin (Sb.26C) was very similar in properties to Fosterite. Optical sensitivity and tensile strength both somewhat lower. The materials for casting the resin were supplied by Scott Bader & Co., Kingsway, W.C.1. Photoelastic models might be cast in their final shape in suitable moulds. Casting techniques still needed improving, and up to the present difficulty was found in casting models whose smallest thickness was much above 2 in., but many models had been cast and investigated by different workers, with completely satisfactory results. Cost worked out at about 8s. per lb.

Fuller details about the properties of most of these materials could be found in any text book on photoelasticity. Further information about Marco Resin could only be obtained at present from those photoelasticians who had been experimenting with it. There were many other materials which exhibited the photoelastic effect, and fresh ones were continually being tested. There was a reasonable hope that even better materials would be found than those at present in use.

DR. HEYWOOD: It seemed to him that in cases where fillets occurred very close to load points in components which had irregular shape, they could not lay down any general law governing how the radius of the fillet would affect the stress; the kinds of stress and the applications of load might be so various that the change in fillet radius might have quite different results between one component and another. He advanced that view as purely his own; he was not an engineer, but a physicist, and consequently in the paper he had tried to keep clear of problems of fatigue, and so forth. He had dealt with the methods and had left it to the engineers to see how they could be applied to engineering problems; probably he would be wrong if he tried to indicate how they should be applied.

Written reply: He was inclined to agree, on further consideration, that his paper might tend to give the impression that the photoelastic method was a more complicated business than it was in practice, in a very large number of cases. He regretted that he had not emphasised more how large was the proportion of very simple cases. The more complicated cases, however, did occur, and he thought that an aircraft designer would be pursuing a very short-sighted policy if he were to ignore the possibility of these and be content to cater only for the simpler type of investigation. The designer knew much better than he what his own problems were, and his object was to show that methods were now available for the solution of many of the more difficult problems which, a few years ago, were quite beyond the scope of any experimental method.

He felt that Dr. Heywood was over-simplifying the problem, however, when he suggested that only the maximum stresses on a free boundary were of interest to the designer. In the case of fatigue stresses at a point of high stress-concentration on a boundary, for example, the extent of the plastic yield must depend upon the rate at which the stress decreased in all directions around the point. It was this factor which was largely responsible for the not uncommon failure of a component at some point other than the point of highest surface stress.

Another example, which was illustrated in the lecture, showed the peak shear-stresses in a bar to occur on the boundary of a hole at a point in contact with a bolt. In that case the maximum tensile stress would probably also occur somewhere on the same loaded boundary, but its location and value were not given directly by the fringe-pattern, but needed the application of one of the integration methods. That type of problem would occur in "three-dimensional" as well as "two-dimensional" cases, and surely it was of sufficiently frequent occurrence, and of sufficient importance to justify (if justification were needed) a mention of the methods by which such contact stresses could be evaluated.

He thought that a discussion on the relative merits of various types of polariscope would be too technical

for this journal. He agreed, however, with Dr. Heywood on the advantage to be gained in many cases by direct observations in a diffused field of light; that was the original method of observation which Filon and he used some thirty years ago. It was still in use for suitable problems at University College Laboratory. The "projection" bench was converted to a "diffused screen" bench, simply by placing a sheet of tracing paper between condensing lens and polariser. Observations were made with a low power travelling microscope.

CAPTAIN PRITCHARD: He had worked with Filon in 1919 and for several years afterwards on photoelasticity, and no one could have a greater admiration than he had for Filon's work. He would have liked to have written more about Filon in the paper. As regards work in the U.S.A., it was of interest to note that Professor Frocht, who had written the classic American work on photoelasticity, had dedicated his book to Coker and Filon and had paid a very high tribute to their work.

MR. HICKSON: He did not emphasise, when he had given the limits of 5-30 per cent. for a three-dimensional model, that that was the range from the simple three-dimensional problem of the surface stresses to the very complicated case of the internal stresses in a non-symmetrical model. The 30 per cent. applied to the latter, and not to a simple surface stress such as could be explored by brittle lacquer; for the simple surface stress the 5 per cent. was nearer the mark. He did not think the consistency of brittle lacquer results had been proved, and he was a little afraid of it.

He was interested in the tentative propositions on the effect of Poisson's ratio, and would like to look into that matter a little more. The biggest effect of Poisson's ratio might be on the smaller stresses; but it would affect also the maximum shear stress, which might be in many cases the relevant factor.

With regard to the surface layer method which was being developed at the College of Aeronautics, he quite agreed that on curved surfaces where stress concentrations occurred they could probably not use a surface layer of photoelastic material sufficiently thin to give anything like a reproduction of the surface strain. But he did think that on flat bars it might give useful information on the occurrence of plastic yield in the materials, and thus could be used for testing materials, though not for exploring stresses.

Written reply: He agreed that he was in error in stating, without modification, that a combination of direct observation through a slice and observation by scattered light would give the required result "most easily." He should have said that that was *one method*, and the one which he had found most satisfactory in some stress concentration problems. He had no intention of implying that any one method or combination of methods would be the best in all cases. In fact he was sure that different individual problems needed different methods, and that frequently one could not know which method of measurement would be most suitable until the frozen model had been examined in a polariscope. He considered, therefore, that it would be unwise for an investigator to limit himself to the use of any one routine.

With regard to Mr. Hickson's tentative propositions on the effect of Poisson's ratio, he felt that there was not yet sufficient evidence to justify the laying down of any rules at all. Those propositions appeared to apply to the two cases referred to: Timoshenko's solution for a spherical cavity in an infinite block under tension, and Neuber's solution for an infinite cylinder with hyperbolic groove.

Another exact solution had recently been produced by R. Capildeo and A. C. Stevenson, who had given him permission to quote results from it before its publication. This was the solution for a sphere under diametral compression along a diameter, and the results for the stresses at the equator and at the centre (the only ones yet evaluated) did not confirm Mr. Hickson's propositions.

(*a*) In both cases the stress in the direction of the load suffered by far the greater change with a change in σ.

(*b*) In the case of the stresses at the centre it was the greater stress which suffered the greater change.

(*c*) The departure from linearity in σ was more marked than in either of the previously considered solutions.

(*d*) The effect of a change in σ was much greater than in the other solutions. The change in the stress-difference on the equator due to a change in σ from 0·25 to 0·5 was 30 per cent. of the greater stress, while that at the centre was 56 per cent. of the greater stress.

It might be noted that in all three solutions σ appeared in each stress in the same form $(a+b\sigma)/(c+d\sigma)$. The departure from linearity would not cause serious error in any of these solutions in the case of interpolation between two values of σ, but errors of up to about 30 per cent. might be introduced if one extrapolated, say to a value of $\sigma=0{\cdot}25$ from results obtained with $\sigma=0{\cdot}35$ and $\sigma=0{\cdot}5$.

While the sphere did not present the case of a stress-concentration at the surface, it was the only one of the three solutions which had completely finite boundaries, and it would, he felt, be rash to dismiss the evidence it provided as inapplicable to stress concentration problems on a boundary.

The suggestion of making a porous plastic which would have a Poisson's ratio of about 0·3 was an interesting one, and he hoped Mr. Hickson would discuss it with the chemists and explore the technical possibilities of suitably "foaming" a resin during casting. Like all the problems on producing and casting suitable resins it would probably be tackled much more efficiently by skilled industrial chemists than by engineers and physicists.

MR. KERRY: Something of the sort suggested by Mr. Kerry could be done and it was the sort of thing that would have to be done by experimenting on models in two or three sections. In one case they could use a bar made of the photoelastic material and the cleat made of

a material which bore to the photoelastic material a suitable strength ratio; and they could do another experiment with the photoelastic cleat and a bar of the other material, to study the stresses there. In his view, that method of approach would be preferable to using different thicknesses of materials. He did not know how the stress distribution might be affected by changes in the relative thicknesses.

In dealing with that problem, no rivets or bolts were used, only pins. If rivets or bolts were put in, the problem would immediately become three-dimensional, which was much more difficult to examine. He assumed that any pressure applied by bolts or rivets would first tend to induce frictional forces and so relieve the load on the rivets themselves; but it would also induce lateral compression, which would increase the total shear stresses around the hole, and that increase of pressure might be liable to cause more trouble than the absence of pressure on the bolts or rivets.

MR. THURGOOD: He did not foresee the possibility of using photoelastic methods for the evaluation of stresses in models of complete wing or fuselage structures. One of the main difficulties, he thought, would occur in the buckling of the thin skins under shear stress long before the applied loads were great enough to produce measurable optical effects in any of the members. In such complex structures maintenance of the exact scale ratio would probably be the only way of ensuring "similarity" between model and prototype, and that would mean that most of the members would be exceedingly thin. He thought that such cases were examples of the type of problem for which the electric strain-gauge method was most suitable.

The "plastics" to which he thought Mr. Thurgood referred were no more plastic in behaviour than were the photoelastic plastics. They consisted, in fact, of some kind of "filling" bound by synthetic resins, and most of the photoelastic resins were originally produced for just such industrial applications. He had not seen any data regarding the stress-strain relations of the various "plastics," but he thought they were likely to have a sensibly linear characteristic. It would be exceedingly useful if a range of photoelastic materials could be found which reproduced the stress-strain behaviour of the various structural materials, but he was afraid that was more than even the most optimistic photoelastician could hope for.

CAPTAIN PRITCHARD, *written reply*: He had left his reply to Captain Pritchard until the last because Captain Pritchard had raised the question which he felt was of the greatest importance to the development of photoelastic applications in this country. He was not sure whether he should take Captain Pritchard's criticism of the lecture as "unsatisfying" as a compliment or the contrary. If he meant that he and his fellow aeronautical experts were left wanting to know more about the subject, then the lecture fulfilled one, at least, of its objects. He hoped, if that were the case, that the desire for further information would not evaporate without leading to definite action. He could assure any would-be enquirers that the photoelastic laboratories at the Universities, as well as those at the R.A.E. and the N.P.L., were always ready to give any assistance they could. If, on the other hand, Captain Pritchard's criticism was on the "indefiniteness" of the information given and the alleged "hedging" on the matter of accuracy, he could only say that he was endeavouring to present a true picture of the position, which could not be done by quoting a single figure for the probable error. An estimate of the order of accuracy to be expected in various types of problem was given later, and this had been further explained in his verbal reply to Mr. Hickson. He felt that no engineer could really expect more, especially in view of the fact that, problem for problem, the accuracy of the photoelastic method in the cases for which it was recommended was generally higher than that of calculations or of other experimental methods.

Captain Pritchard deplored the slowness with which photoelastic methods had been taken up in industry, and suggested that University research departments and Industry were equally to blame. He disagreed with him. The responsibility for lack of active co-operation rested squarely on the shoulders of Industry. During the years 1912 to 1930, Coker lectured on photoelasticity to all the Engineering Institutions and to many large industrial concerns. In 1918, as Captain Pritchard recalled, Major Low lectured to the Royal Aeronautical Society and called attention to the need to subsidise and encourage research in the subject. What happened? The engineers, almost without exception, expressed their belief in the potentialities of the new method—and sent the few pioneers back to their laboratories (probably wishing them "the best of luck") and saying they would be interested to hear of any new developments! If, during these years, Industry had shown any initiative, if from their large resources, they had done what Industry did in America after Coker's lecture tour there—established research units to hasten and further the development of the work—then in all probability those discoveries which eventually took place in America in the late 1930's would have taken place in this country, and moreover research here would not have ceased temporarily on the deaths of Filon and Coker.

It was a disaster that Filon's death and Coker's final illness occurred almost simultaneously just before the war. Many of them were away during the war, serving in some technical capacity or other in the Forces, and the staffs who remained with the Universities had no time for research. Industry, however, was more vitally interested than ever in stress problems, and was in a position to make use of and develop the new ideas from America. Some few large firms did, in fact, keep in touch with new developments and did photoelastic work on their own particular problems, but in general there was no sign that Industry was interested in the subject. Finally, it was left to the Universities after the war to effect co-operation between workers in this field by the formation of the Stress Analysis Group which was later affiliated to the Institute of Physics, and, be it noted, among all the Institutions and industrial concerns invited to co-operate, the Institute of Physics was the

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only one which made any offer of financial or other assistance !

(It was not inappropriate to mention here that this Group, with a membership from academic, government and industrial research departments, was concerned with all forms of experimental stress-analysis, and its function was mainly the exchange of information and ideas on new methods and techniques. It was probably the best means by which stress-analysts could keep in touch with current work all over the country, and the Group would welcome still more members from Industry.)

The position today was in many respects similar to that in the 1920's, for although there were now many more workers in this field there were still far too few and there was much more to be done. Two results followed from that—industry as a whole, and perhaps the Aircraft Industry especially, was not getting the advantages which photoelasticity offered; and photoelastic methods were not receiving the impetus to their development which only work on a wide variety of problems could give.

Captain Pritchard suggested that this had been an opportunity to "put the subject across in a big way to aeronautical engineers." He felt that the photoelastic method itself was already accepted in principle by practically every branch of the Aircraft Industry, and needed no "putting across." He would, however, act on the spirit of his suggestion by endeavouring to emphasise what were the relative roles of the academic and the industrial research establishments in the development of the subject.

THE UNIVERSITIES AND TECHNICAL COLLEGES HAD TWO MAIN FUNCTIONS:—

(1) THE DEVELOPMENT OF THEORY AND NEW METHODS AND THE DEVISING OF SUITABLE TECHNIQUES.

(2) THE TEACHING OF THE SUBJECT AT UNDERGRADUATE AND POST-GRADUATE LEVELS.

With those two functions the whole of the resources of those institutions in staff, technicians and workshop and laboratory facilities were fully occupied. During the course of their work it was possible, and even necessary, from time to time, for the departments to work on some particular engineering problem, but the investigation of such problems was not their job. On more than one occasion at the photoelastic laboratory at University College they had been asked if they could undertake comprehensive investigations to obtain data which could be made available to industry and which might, in some case, form a basis for the consideration of one of the British Standards Committees. Neither staff nor students would be justified in wasting their time on investigations of this kind which, of necessity, were largely routine work. Was not this a matter which should be dealt with by the establishment of a testing laboratory by some organisation such as the D.S.I.R.? They already had the nucleus of this in the well equipped photoelastic research laboratory at the N.P.L. This might easily be expanded by the addition of more apparatus, and there would be little difficulty in training a staff of laboratory assistants who could carry out the work under direction.

From the standpoint of teaching, also, the position was probably not generally understood by Industry. In the first place, a course of lectures on photoelasticity, or the reading of a book on the subject, was of little use unless accompanied by practical work by the student. The provision of enough photoelastic equipment to give students the bare minimum of practical work was probably the most serious bottle-neck in the training. Unless a laboratory were to confine its post-graduate teaching to one or two students a year, it must be content to give short courses, which, to be effective, must be followed closely by much more extended practical work. That was where action was needed on the part of Industry. It was not enough to send a man to a vacation course, and then to retain him as a "photoelastician, unemployed" until such time as it was decided to install a photoelastic laboratory. That, in fact, was what had happened to too many of the men who had attended the courses at University College. Their object in these short courses was to give sufficient training to enable a man to undertake the establishment of a laboratory and the carrying out of stress-explorations. In the early stages he might have to seek advice from time to time but he could still be doing useful work, and it was of primary importance that he should be given the facilities to acquire experience.

THUS THE FUNCTION OF INDUSTRY WAS TO PROVIDE THE FACILITIES FOR THE CONTINUED TRAINING OF THEIR PHOTOELASTICIANS AND FOR THE SOLUTION OF THEIR OWN PROBLEMS.

In the Aircraft Industry a few firms already had a photoelastic laboratory in use. Some were in process of installing one, others, he understood, were considering the matter. That was progress in the right direction, but it was not enough. Surely such a large and important industry could do something in the way of co-ordinating effort. Could not the S.B.A.C. establish a central laboratory where problems common to the whole industry could be investigated and where a certain number of men could gain experience ?

To sum up: The scientists had worked out the theories, evolved the methods and devised techniques. All those had been tested, and found capable of being of immediate value to industry. For the improvement and development of methods and techniques, and for the extension of the field of application, what was needed was a large increase in the number of men working on industrial problems. It was up to Industry to move in the matter and, he thought, the move must come from the heads of Industry; from the men who were in a position not to be afraid of taking up something new. Progress would not be made without their authority and support.

The Resistance to Air Flow of Wire Gauzes

by

W. J. D. ANNAND, B.Sc. A.F.R.Ae.S.
(Rolls-Royce Ltd.)

SUMMARY:—Wire gauzes are used in air duct systems and wind tunnels for many purposes. Methods of estimating the pressure loss through a gauze currently available are often found in practice to be unsatisfactory. In this paper data obtained by several experimenters are correlated. It is shown that the effect of Reynolds number on pressure drop is similar for a large range of gauze porosities. An empirical method of estimation of the pressure drop is given which can be used with confidence for gauzes of porosity (defined as the proportion of area not blocked by the wires) from 0·27 to 1·00, and over a range of Reynolds number, based on approach velocity and wire diameter, from 20 to 600.

1. Introduction

Wire gauzes are used in air duct systems and wind tunnels for a variety of important duties. Some examples are:—

(*i*) to catch stones or other injurious material, as in engine air intakes.

(*ii*) to produce uniformity of the velocity distribution across a duct, as in many wind tunnels and other test apparatus. Non-uniformities are reduced because the gauze presents greater resistance to flow where the velocity is higher. Refs. 2 and 7 present some theoretical considerations and experiments on this application. The use of gauzes in diffusing ducts presents special features in that the effect of the gauze penetrates upstream, reducing the causes of non-uniformity as well as the non-uniformity itself. Extensive experiments are reported in Ref. 3.

(*iii*) to reduce the turbulence of a stream. Much use is made of fine gauzes for this purpose in low-turbulence wind tunnels. The components of turbulence are modified in the contraction on entering the gauze and, provided that the gauze wires are fine and the porosity suitably chosen, the overall turbulence some distance downstream can be considerably reduced below that in the approaching stream. Experiments on this effect are reported (for example) in Ref. 2, and Ref. 7 gives some theoretical considerations.

(*iv*) oppositely, to produce artificially high turbulence. For this purpose, coarse grids of relatively large diameter rods are used; the large eddies in the rod wakes presumably produce by their breakdown the turbulence required.

(*v*) to introduce known pressure drops into experimental systems.

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For all these applications it is desirable to be able to estimate accurately the pressure drop which will be produced by a given gauze—particularly so when the gauze is to be used to reduce the non-uniformity of a stream, since in this case[7] a certain definite value of the ratio of pressure drop to approach dynamic head ($\frac{1}{2}\rho V^2$) is necessary for the most complete removal of the irregularities. Nevertheless experience shows that the empirical formulae usually employed are often considerably in error. Generally speaking, these have been based on very limited data, and it is the purpose of the present paper to review and correlate the available information and to derive an empirical relation which can be used with confidence throughout a wide range of conditions.

Notation

a	coefficient in the formula for the estimation of k (*see* Section 5)
d	mean wire diameter
D_1, D_2	"depths of weave" in the two directions (*see* Fig. 1)
l_1, l_2	pitches in the two directions of the weave (*see* Fig. 1)
k	pressure drop coefficient = pressure drop/$\frac{1}{2}\rho V^2$
R	Reynolds number = Vd/ν
β	porosity
V	approach velocity
ν	kinematic viscosity

2. Definitions

The main dimensions describing the geometry of a gauze are the mean wire diameter d, and the pitches l_1 and l_2 in the two directions of the weave (*see* Fig. 1). From these can be calculated the porosity β, defined as the area of the holes in a unit area of gauze, as projected orthogonally onto a parallel plane. They are related by $\beta = [1-(d/l_1)][1-(d/l_2)]$.

Two other dimensions characterising the gauzes are the "depths of weave" D_1 and D_2 in the two directions. Fig. 1 makes clear the meaning of this term. Only one investigator has measured these dimensions for the gauzes tested. Although they might be expected to have some influence on the flow pattern through the gauze, the indications are that the effect on pressure loss is very small for gauzes of normal manufacture.

Any one of the gauze dimensions can be used to form a Reynolds number characterising the flow. The diameter d has been chosen, giving a Reynolds number $R = Vd/\nu$, where V is the approach velocity and ν the kinematic viscosity. It will be shown later that this choice gives good correlation.

The pressure drop is defined as the loss in stagnation pressure from a plane immediately upstream of the gauze to one sufficiently far downstream for the wakes of the wires to have become indistinguishable by mixing—that is, about 30 wire diameters downstream, to judge from certain measurements on a coarse grid reported in Ref. 4. In practice the pressure drop is usually measured by the difference in static pressure between planes some distance upstream and downstream of a gauze sample in a duct of constant section.

The pressure drop coefficient k is defined as the ratio of this loss to the approach dynamic head $\frac{1}{2}\rho V^2$.

3. Sources of Data

Test results have been drawn from four sources, in all of which each gauze was tested at a number of air speeds.

(*a*) Eckert and Pfluger[5] tested 16 commercial gauzes, measuring the loss of static pressure between tappings about 5 cm. upstream and downstream of the gauze in a tube of 45 cm. diameter. Their results are presented only in graphical form, and it is difficult to pick off the individual test measurements accurately. For this reason only the results for four gauzes have been made use of for analysis (unfortunately, not including the gauze of greatest blockage; the dimensions quoted for this are inconsistent).

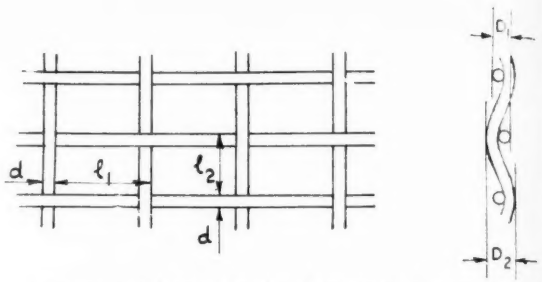

FIGURE 1. Diagram showing characteristic dimensions of a gauze.

(*b*) Taylor and Davies[6] tested four gauzes, measuring the loss of static pressure between tappings about 2 in. upstream and downstream of the gauze in a tube of about 3·2 in. diameter. Their results are tabulated and the measuring accuracy is good.

(*c*) Simmons and Cowdrey[4] tested eight gauzes, measuring the loss of static pressure between tappings 6 in. upstream and downstream of the gauze in a 6 in. square duct. Their results are tabulated and the measuring accuracy is very good.

(*d*) Schubauer, Spangenberg and Klebanoff[2] tested 10 gauzes, using a rather different arrangement from

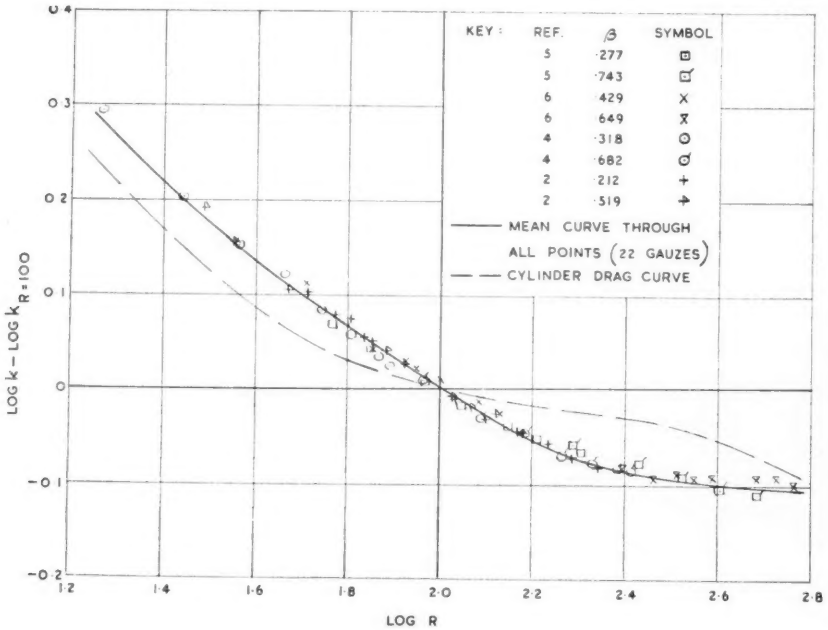

FIGURE 2. Representative selection of experimental points showing correlation.

any of the foregoing. The gauzes were fastened across the end of a square duct and air was blown through them into the open atmosphere. The pressure drop was measured as the difference in pressure measured by static probes placed centrally upstream and downstream of the gauze. Here again the results are given only graphically on a rather small scale, and the accuracy with which points could be picked off is rather low. Results for six of the gauzes were used in the analysis.

The measurements include the pressure drop due to wall friction over different lengths of ducting. It is impossible to estimate this loss accurately because the presence of the gauze completely alters the velocity profile near the wall, both upstream and downstream. However, it is improbable that the correction to the measured pressure drop coefficients would exceed 0·03, so that the omission of the correction is not of importance.

The dimensions of the gauzes included in the analysis are given in Table I. Most are of square mesh or nearly so; the greatest deviation from squareness is $l_2 = 0{\cdot}80\, l_1$.

4. Variation of Pressure Drop Coefficient with Reynolds Number

The first step in the analysis was to plot all the pressure drop coefficients logarithmically against Reynolds number based on the wire diameter. It was soon noticed that the forms of the curves obtained for the various gauzes were very similar, and on investigation it was found that identical curves, shifted only along the log k axis, could be fitted to all; or alternatively, by shifting each set of points parallel to the log k axis, all the points could be brought onto a single curve. This is illustrated in Fig. 2 where (log k − log $k_{R=100}$) is plotted

TABLE I

DETAILS OF GAUZES TESTED

Source	Wire diam. d : in.	Pitches l_1 : in.	Pitches l_2 : in.	l_2/l_1	Weave depths D_1/d	Weave depths D_2/d	Porosity β
Ref. 5	0·0059	0·0125					0·277
	0·0055	0·0154					0·413
	0·0098	0·0547					0·672
	0·0118	0·0856					0·743
Ref. 6	0·0087	0·0251	0·0251	1·00	1·25	2·61	0·429
	0·013	0·0417	0·0417	1·00	1·53	2·23	0·475
	0·020	0·0850	0·0850	1·00	1·94	2·15	0·585
	0·024	0·126	0·121	0·96	1·89	2·05	0·649
Ref. 4	0·0044	0·0103	0·0099	0·96			0·318
	0·0053	0·0126	0·0125	0·99			0·333
	0·0068	0·0167	0·0167	1·00			0·351
	0·0067	0·0192	0·0166	0·86			0·389
	0·0107	0·0333	0·0318	0·95			0·451
	0·0075	0·0250	0·0249	1·00			0·490
	0·0072	0·0417	0·0333	0·80			0·649
	0·0147	0·0869	0·0832	0·96			0·682
Ref. 2	0·0135	0·025					0·212
	0·0170	0·050					0·436
	0·0055	0·0185					0·494
	0·007	0·025					0·519
	0·0075	0·042					0·672
	0·025	0·250					0·810

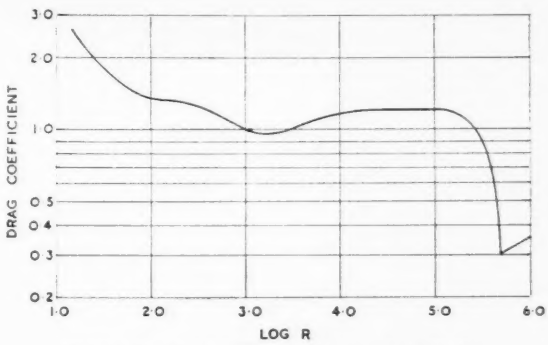

FIGURE 3. Drag coefficient of a circular cylinder.

against log R for two gauzes from each source. In the complete plot, individual points would be indistinguishable near the mean line; of 178 points representing tests on 22 gauzes, only 16 fall away from the mean line by more than 3 per cent. of k, and of these 16 all but five are points picked off from the graphs of Refs. 5 and 2.

This degree of correlation is quite striking, and in some respects was unexpected. Some variation of the shape of the Reynolds number curve with β was expected, because the variation of k with R probably arises partly from variation of the discharge coefficient of the orifices formed by the meshes, and this variation would be expected to depend on the contraction ratio of these orifices—that is, on $1/\beta$. Further, it would be expected that at very high β the variation of the loss coefficient with R would approach that of the drag coefficient of an infinite circular cylinder; this curve is plotted in Fig. 2 for comparison and it will be seen to differ somewhat from the curve for k. Nevertheless, no significant alteration of the curve shape with β could be detected over the range covered ($0{\cdot}21 < \beta < 0{\cdot}81$).

Again, in Ref. 2 observations are reported on the eddy formation behind several gauzes. It was found that at very low Reynolds number regular eddy formation behind the wires of a gauze did not occur; at higher Reynolds number eddies were cast off with regular periodicity. These two states presumably correspond to the observed flow pattern changes round isolated circular cylinders for which, at low Reynolds numbers, two vortices form just behind the cylinder and remain there, while at higher Reynolds numbers vortices are cast off alternately from opposite sides, forming the well-known "Karman trail." It was found in the tests of Ref. 2, made on six gauzes, that the "eddy critical" Reynolds number, at which the change-over from one regime to the other took place, varied directly with β. This correlation would be expected to apply also to the variation of k with R—that is, the curves of log k would be expected to be similarly positioned when plotted against log (R/β) instead of log R. However, plotting shows that this is not the case (*see* Fig. 2).

The overall variation of loss coefficient over the range of Reynolds number covered is 2·5 : 1. By analogy with the drag coefficient curve for an infinite cylinder, the log k curve may be expected to continue to rise as log R is further reduced, and it may also be expected

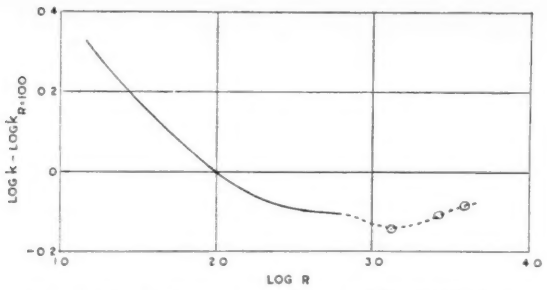

FIGURE 4. Possible extension of curve of Fig. 2 to higher Reynolds numbers.

that large variations may occur at considerably higher Reynolds numbers, despite the fact that the curve flattens out at the upper end of the range covered. In Fig. 3, the cylinder drag curve is plotted over a wider range of Reynolds number, and it will be seen that after flattening out in the range of R between 100 and 250, the curve drops again at about 350, then rises, levels off, and finally drops sharply at $R \simeq 2 \times 10^5$. Some idea of the actual trend of the gauze pressure drop curve at Reynolds number up to about 4,000 may be obtained from three measurements made by Simmons and Cowdrey[4] on a grid of rods 0·25 in. diameter at 0·9 in. pitch. The observed values of k are given in Table II.

TABLE II

MEASUREMENTS ON A COARSE GRID OF 0·25 IN. DIAMETER RODS[4], ($\beta = 0{\cdot}522$)

Air speed V ft./sec.	R	k
10	1310	1·34
20	2620	1·45
30	3930	1·50

In Fig. 4, $\log k - \log k_{R=100}$ is plotted against $\log R$; the method given in the next Section was used to estimate k at $R = 100$. A curve somewhat similar to the cylinder drag curve can be used to link these points to the mean curve established up to $R = 600$, as shown by the dotted line.

5. Variation of Pressure Drop Coefficient with Porosity

Using the general curve of Fig. 2 to extrapolate where necessary, it is possible to pick off for each gauze the value of k corresponding to a given Reynolds number. Table III gives the values so obtained at $R = 100$.

Preliminary plotting showed that the values of k so obtained from the results of Ref. 2 were consistently lower than the others. It seems probable that this is a genuine difference arising from the different experimental arrangement used. Absence of constraint on the downstream side of the gauze may have affected the flow pattern, but it seems unlikely that this could have had much effect near the centre of the gauze; a more likely explanation is that the method of measurement of the pressure on the downstream side of the gauze was at fault. The gauze under test was supported on flanges attached to the duct outlet; it is known that a jet issuing from a duct with a flanged outlet into still air produces a small depression across the outlet, and it is probable that this depression is not uniform across the section. These results therefore were excluded from the investigation of the variation of k with β.

TABLE III

MEASURED VALUES OF k AT $R = 100$

Source	β	k	$0{\cdot}71\,(1-\beta^2)/\beta^2$
Ref. 5	0·277	8·22	8·53
	0·413	4·06	3·46
	0·672	0·81	0·86
	0·743	0·57	0·58
Ref. 6	0·429	3·31	3·15
	0·475	2·45	2·44
	0·585	1·47	1·36
	0·649	0·79	0·98
Ref. 4	0·318	6·10	6·32
	0·333	5·66	5·68
	0·351	5·12	5·05
	0·389	4·12	3·99
	0·451	2·99	2·78
	0·490	2·24	2·24
	0·649	0·79	0·98
	0·682	0·81	0·81
Ref. 2	0·212	13·21	
	0·436	2·74	
	0·494	1·71	
	0·519	1·45	
	0·672	0·72	
	0·810	0·38	

Some assistance in deciding on the form of the relation between k and β to be used as a basis for analysis can be obtained from simple theoretical ideas. Consider a stream filament which just passes through a single mesh. It suffers first a contraction down to the "throat" of the mesh, during which its mean velocity is increased to V/β; if it is assumed that losses during this process are negligible and that the stream breaks away from the wires at this section and then expands freely to the original area, the stagnation pressure loss during this process being that corresponding to a sudden expansion through the area ratio $(1/\beta)$, it is easily shown that the loss coefficient k is equal to $(1-\beta)^2/\beta^2$. In practice, the flow pattern will not be as simple as this. There will be friction losses at the wire surface, with the formation of boundary layers which will increase the effective contraction, and the point of breakaway probably moves round the wire as Reynolds number varies. However, it seems reasonable to investigate first the suitability of

TABLE IV

ISOLATED MEASUREMENTS FROM REF. 1

β	Test R	k	k corrected to $R = 100$	$0{\cdot}71\,(1-\beta^2)/\beta^2$
0·372	102	4·67	4·68	4·41
0·469	165	2·10	2·37	2·52
0·554	252	1·20	1·47	1·60

TABLE V

VALUES OF a FOR VARIOUS VALUES OF R

R	20	25	30	40	50	60	70	80	90
a	1·30	1·18	1·09	0·97	0·89	0·84	0·80	0·76	0·73

R	100	200	300	400	500	600
a	0·71	0·60	0·57	0·56	0·56	0·56

the slightly more general formula $k=a(1+b\beta+c\beta^2)/\beta^2$. If the formula is to be usable for gauzes of high β, it must give the value $k=0$ for $\beta=1{\cdot}0$, corresponding to the empty duct. The numerator must therefore contain a factor $(1-\beta)$, and it may be written $k=a(1-\beta)(1-c\beta)/\beta^2$. By plotting $k\beta^2/(1-\beta)$ against β, it was found that the best mean curve of this form is given by

$$k=\frac{0{\cdot}68\,(1-\beta)(1+1{\cdot}1\,\beta)}{\beta^2}.$$

Calculation showed, however, that over the range of the tests ($\beta \not< 0{\cdot}27$) the values of k given by this formula are not significantly different from those given by the simpler equation

$$k=\frac{0{\cdot}71\,(1-\beta^2)}{\beta^2}.$$

In Tables III and IV and Fig. 5, this relation is compared with the measurements, including some isolated measurements from Ref. 1 not used in deriving the empirical formula. Agreement is quite good. The curve falls slightly high at β less than 0·35 and slightly low for β between 0·35 and 0·45. Better agreement could probably be obtained by fitting a more elaborate equation containing higher powers of β, but it has not seemed to the author that the accuracy of the experimental data would justify this elaboration.

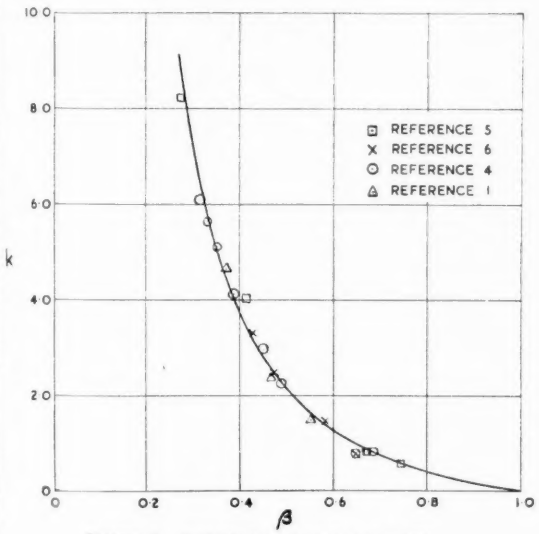

FIGURE 5. k plotted against β at $R=100$. Curve drawn is $k=0{\cdot}71\,(1-\beta^2)/\beta^2$.

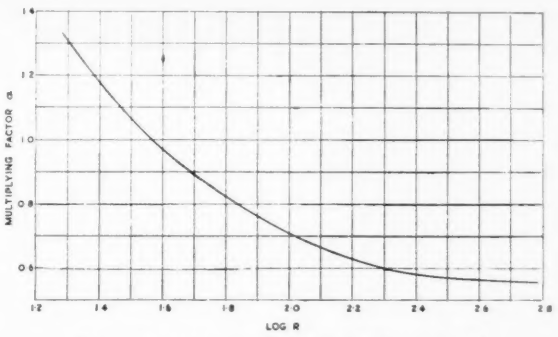

FIGURE 6. Variation of multiplying factor a with Reynolds number.

Some indication that the formula can be used down to $\beta=0{\cdot}21$ is given by the fact that a similar formula with a lower coefficient fits the results of Ref. 2 satisfactorily down to this value.

6. Working Method of Estimation of k

The working formula for the estimation of the pressure drop coefficient is then

$$k=\frac{a(1-\beta^2)}{\beta^2},$$

where a depends on the Reynolds number based on wire diameter, as shown in Fig. 6 and Table V.

To assist in practical application, the nomogram given in Fig. 7 has been prepared. From this chart the multiplying factor a can be obtained directly from the values of the wire diameter, air speed, air temperature and air pressure. The method of use is indicated by the dotted lines; join diameter and air speed and find intersection on reference line I; join air temperature and pressure and find intersection on reference line II; join the two intersection points and read off a. The point on reference line II corresponding to I.C.A.N. standard sea level conditions is ringed.

It should, perhaps, be emphasised once more that the curve of variation of a with Reynolds number should not be extrapolated outside the range of the test data. The formula can be used with confidence for β not less than 0·27 and is probably adequate down to $\beta=0{\cdot}20$; below this value, there is no evidence as to the adequacy of the formula, and testing would be advisable.

One other point concerning practical application should be remembered. That is, that accurate estimation of β and of k at low values of β calls for very accurate measurement of the gauze dimensions. Table VI gives the percentage error in k produced by a one

TABLE VI

EFFECT ON ESTIMATE OF k OF ONE PER CENT. ERROR OF WIRE DIAMETER

β	0·2	0·3	0·4	0·5	0·6	0·7	0·8
Percentage error of estimated k	5·2	3·6	2·8	2·2	1·8	1·5	1·3

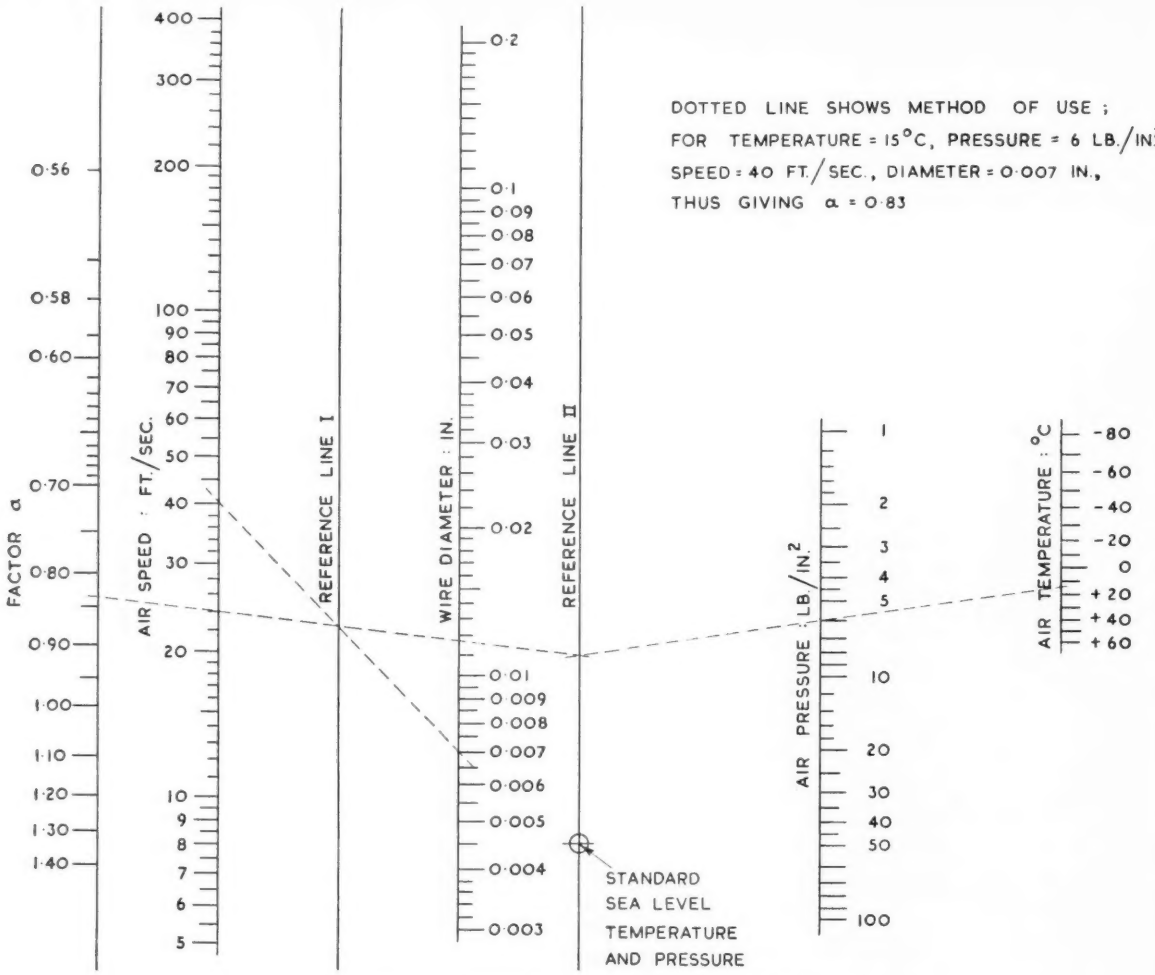
DOTTED LINE SHOWS METHOD OF USE ;
FOR TEMPERATURE = 15°C, PRESSURE = 6 LB./IN.², SPEED = 40 FT./SEC., DIAMETER = 0·007 IN., THUS GIVING α = 0·83
FACTOR α
AIR SPEED : FT./SEC.
REFERENCE LINE I
WIRE DIAMETER : IN.
REFERENCE LINE II
AIR PRESSURE : LB./IN.²
AIR TEMPERATURE : °C
STANDARD SEA LEVEL TEMPERATURE AND PRESSURE

FIGURE 7. Nomogram for determination of multiplying factor a.

per cent. error in measurement of the wire diameter d at various values of β. At $\beta = 0{\cdot}3$, for example, one per cent. error in d produces 3·6 per cent. error in k. Now one per cent. error in d may, for the wire sizes commonly used in gauzes, be less than 0·0001 in., so that the need for accuracy is evident.

Provided that the limitations are observed and the necessary accuracy of gauze measurement obtained, the method, according to a rough statistical estimate, should give the pressure drop within 20 per cent. on 95 per cent. of occasions.

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Experiments With Flat Planes

(1866-1890)

by

J. LAURENCE PRITCHARD

(From time to time we hope to be able to publish extracts from the notes for the History of the Royal Aeronautical Society now being prepared by Captain Pritchard)

THE PIONEER rarely receives that meed of praise which is his due, until those days arrive when his pioneering has become a page or so of history. What meed of praise he may then receive is too often made savourless by critics unable to disassociate themselves from the knowledge of their own technical era. The halting speed of pioneering aerodynamic research cannot be judged by the supersonic speed of secret splittings of the atom.

It was R. L. Stevenson who said that to travel hopefully is a better thing than to arrive. In aviation we have arrived at that streamline flow of technical jargon which deludes so many they have at last reached the boundary layer of aeronautical knowledge, so that they find experiments with plates always a little flat, although their curiosity may still be aroused by flying saucers.

Eighty-seven years ago there were many members of the Aeronautical Society of Great Britain who were travelling hopefully along roads which were badly lit by the first few glimmers of the dawning of science. And what fun they had in their Victorian travels on those dimly lighted roads! And what fun an amateur historian must have trying to travel for a little while with them before the roads are completely obliterated by the jargon of jets and the moonshine madness of interplanetary travelogues.

Eighty-seven years ago? That was the year the Society was founded, 1866. Those who became members then had travelled alone, but from that year they were able to travel in company.

It was in the 1850's that the great W. H. Wenham travelled up the Nile and there studied, from the point of view of an engineer, the slow flight of that unbelievable bird, the pelican; and that graceful ballet dancer in the air, the spoonbill; and the flight of a dozen other birds indigenous to the Egyptian air. On his return he wrote, in 1859, that classic paper which saw the light of day when he read it at the first lecture meeting in 1866 of the newly formed Aeronautical Society of Great Britain. In his paper, entitled "Aerial Locomotion and the Laws by which Heavy Bodies impelled through the Air are Sustained," Wenham remarked on the importance of a rigid leading edge; on the fact that the greatest part of the lift on a plane was in the first eighth to tenth part of the wing; and that man could never hope to fly under his own power. He advocated the long narrow plane, and to avoid great spans suggested the use of super-imposed planes.

Wenham, during the discussion following his lecture, drew attention to the work of Sir George Cayley, to whom as early as 1846, twenty years before the foundation of the Society, W. S. Henson had written on behalf of himself and John Stringfellow, "We therefore resolved to apply to you as the Father of Aerial Navigation . . .", a description which has been more and more emphasised during the century which has followed, as the work of Cayley has become more widely appreciated. Wenham reminded his listeners that Cayley had in 1810 drawn attention to the movement of the centre of pressure at high speeds and small angles of attack.

Indeed, Cayley, Wenham and others were well aware of the kind of data that was required, but experiment and research were painfully slow. F. W. Lanchester in his *Aerodynamics*, published in 1897, gives a brief account of some of the earliest attempts at aerodynamic research. To quote, "The earliest experimenters, Robins (1761), Hutton (1787) and Vince (1794-95, 1797-98) employed a primitive form of whirling table, the invention of which is attributed by Hutton to Robins."

These results and experiments of others were known to the early members of the Society but they led, in certain respects, to errors of belief about the pressures on flat plates at various angles of attack. Over a hundred years after Robins, the problems raised by flow of air over plates and bodies of different shapes remained unsolved and on 24th August 1866 the Council passed the following remarkable resolution:—

"It was delegated to the Duke of Argyll, Lord Dufferin, Mr. James Nasmyth, Mr. J. McConnell, Mr. Glaisher and Mr. Wenham, to consider the law of resistance of inclined surfaces moving in elastic and non-elastic fluids, as air and water, and the resultant force obtained at right angles to the direction of motion. Experiment has now proved that as the velocity is increased, and the angle of inclination diminished, the centre of effort of the plane approaches forward, till nearly the whole effect of reaction or support is transferred to the front edge. It therefore follows that in very acute angles with the line of motion, a given extent of reacting surface at high velocities will be most effective if disposed in a narrow plane extended transversely to its direction.

"An investigation of this principle will determine the best ratio of surface to speed in the long narrow wings

To the Council of the Aeronautical Society of Great Britain.

Report of Messieurs Moy and Jay, Members of the Aeronautical Club, as to experiments to be made at the expense of the Society, to ascertain various facts as to the action of aeroplanes in motion.

See accompanying Plan.

It is proposed to construct a suitable stand, with an iron or steel vertical axle and driving pulley keyed thereto, and radial arms of 14 feet each.

A pulley and crank to raise a weight and driving band

A counter pulley and balance weight to keep band taut.

The diameter of the orbit will be 28 feet, and the calculation will therefore be one mile per hour for each revolution per minute.

The experiments to be carried out will consist of:-

1 Lifting power of mounted planes at various angles and speeds.

2 Benefit to be derived from superposed aeroplanes

3 To ascertain the actual working centre of effort or pressure

4 The best proportion, as to length and breadth of aeroplane surfaces, for maximum of effect with minimum of resistance and weight

And such other experiments as may be suggested

It is estimated that the whole cost will not exceed the sum of £15-0-0

Thos. Moy.

R. C. Jay

approve F. H. Wenham

13 Aug 1868

FACSIMILE of Moy's proposal to the Council for experiments on the "behaviour of aeroplanes in motion."

of birds of extended flights; in the blades of reciprocating and screw propellers; and in the sails of vessels for sailing near the wind."

The resolution was not acted on as a matter of urgency. After all, it was the Victorian age, and there were more important things to do than to follow the flying will-o'-wisp! Some idea of the attitude of mind generally of the public at the time, an attitude which inevitably had some effect on how urgent the suggestion might be, may be obtained from the words used by Stewart Harrison on 15th July 1867. He was proposing a vote of thanks to the Duke of Argyll and Mr. Hurry for taking the Chair at the lecture meeting of the Society.

"I have no doubt," he said, "that the people outside thought it very desirable that those present should be taken care of by their friends. I do not, but I do think that, as gentlemen who risked something of their reputation for soundness of judgment by taking the Chair when such flighty subjects were discussed, those present could not do less than acknowledge their kindness by a vote of thanks."

Meetings were not so frequent in those days as they are now, nor were the opportunities there for pressing forward with ideas about flying. However there were members of the Society always ready to keep the objects of the Society in the foreground on those annual occasions when a lecture was held. At the May meeting in 1869 a Captain Murray, R.N., said he thought the Society should offer a prize of £100 "for a complete and general solution, on rigid mathematical principles" of the following problem:—"Required the relation between the velocity of a current of air and its pressure on a plane surface of given size, shape and inclination." Captain Murray offered to subscribe £10 towards the fund.

At the same meeting a member, E. W. Young, said he had made some rough experiments upon the lifting effect of the wind upon an inclined surface. He had found that with a vane inclined at an angle of 12° to the horizontal the vertical lifting effect was about one half the pressure which it exerted against the same vane placed vertically.

Almost exactly a year later the Council discussed again the absence of data and resolved that an Experimental Fund be set up and an Experimental Committee be appointed. In the Annual Report for that year, 1870, it was stated, "The first great aim of the Society is the connecting of the velocity of the air with its pressure on plane surfaces at various inclinations," and continued, "The Aeronautical Society has been endeavouring for some time to inaugurate a systematic series of experiments on the connection between the pressure and velocity of air, and it is believed that these will afford the only data on which a true science of aeronautics can be founded. For this purpose an instrument has been designed by Mr. F. H. Wenham, and approved by the Experimental Committee, which it is intended to submit to the action of a fan not less than 30 in. in diameter, capable of delivering about 3,000 cubic feet of air per minute. A clear space of 15 feet or more in front of the fan will allow room for a square wooden tunnel to guide the blast, ascertain its velocity, and insert the anemometer. Mr. John Browning, F.R.A.S., will undertake the construction of the instrument, and Sir William Fairbairn, F.R.S., will afford all the aid in his power for its submission to the required test."

These were the first experiments to be carried out by the Society, and the first wind tunnel experiments for aeronautical purposes in the world. Sir William Fairbairn was a famous engineer and, like Browning and Wenham, a member of the Council. The experiments were carried out at the engineering works of Messrs. Penn at Greenwich.

The results of these experiments of over eighty years ago, were published in the Annual Report of the Society for 1871. They were not very satisfactory, for the sources of error were great and largely unknown to the experimenters. Angles of incidence below 15° were not tested and the conclusion was drawn that the lift varied with the sine of the angle of incidence. But from these experiments the Council believed that the power for flying was not so great as had been expected. The horse power of many of the larger birds had, indeed, been exaggerated. Wenham always said that the strength of a bird was less than many had asserted and said in his 1859 paper of the pelican, "it displays no symptom of being endowed with great strength, for, when only slightly wounded, it is easily captured, not having adequate power for effective resistance, but heavily flapping the huge wings, that should, as some imagine, give a stroke equal in vigour to the kick of a horse."

"It is desirable that these important experiments should be verified and continued," added the Annual Report, and continued, "Up to the present time only flat surfaces have been experimented upon, and as all the sustaining surfaces in the wings of birds are curved or hollow, it remains to be proved what is the relative advantage of this form, for it is upon such data that all plans of construction must be based, and on which failure or success must depend, and to determine whether flight is practicable or not."

At the Annual Meeting held in 1872, James Glaisher, from the Chair, drew attention to the difficulties of measuring lift and drag and said, "Anyone who had not considered with some care the nature of the pressure produced by the flow or rush of a fluid, elastic or incompressible, against a plane surface placed in its course, might imagine that the system of parallel forces was merely equivalent to a single resultant force acting at the centre of pressure, and capable of resolution according to the ordinary parallelogram law. But this of course is not the case, for the particles of the fluid which come in contact with the plane have somehow or other to get out of the way, by gliding along the surface of the plane (as they cannot get through it) and this produces a complication in the neighbourhood of such a kind as cannot theoretically be predicted. One thing, however, is quite clear, and that is that the directions of all the small forces acting on the surface certainly are not parallel, and we must therefore have recourse to experiment."

The problem of the boundary layer and the difficulties of turbulent flow were then foreshadowed.

Research without the powerful incentive of the spirit or of necessity moves but slowly. The experiments of Wenham and Browning had attracted considerable attention, and Thomas Moy who, later, was himself to continue the experiments, said at the 1874 meeting that he hoped some clever geometrician or mathematician would construct a formula from Wenham's results and demonstrate the true law of air resistance. At that date mathematicians had not been so well tamed as they are now.

In May 1877 Thomas Moy and another heavier-than-air enthusiastic member of the Society, R. C. Jay, appealed to the Council for a grant from the Experimental Fund for apparatus for further experiment. It was not until a year later that their proposal was approved by Wenham on behalf of the Council and a grant of £20 made from the Fund. The proposal, in Moy's own handwriting, is reproduced on page 148. It is one of the treasured possessions of the Society. It will be noticed that Moy estimated the cost at £15 and the Council granted £20. Financial control, international, national or local had not then raised its octuple tentacles to curtail the enterprise of the perishing pioneer.

Moy's report was submitted to the Council on 4th April 1879. It read as follows:—

"The apparatus answers its purpose very well up to a speed of 21 miles an hour. Beyond this speed it is very difficult to turn the handle fast enough; and it will be necessary to have a smaller grooved pulley on the upright shaft in order to increase the speed and obtain pressures above 20 miles an hour.

"The series of experiments tried during the autumn and winter of 1878 were for the purpose of finding the centre of pressure upon inclined rigid planes, as it had always been found, in practice, that the centre of pressure approached the front edge of such planes when acted upon by the air.

"Numerous planes were tried, and various expedients adopted in order to work out this very delicate operation. At last, after many attempts and failures which I need not describe, a thin wooden plane 4 inches wide by 12 inches long was mounted with an axle which could be made to compensate for any unequal weight existing between the fore and aft portion; a loose pin held it at the angle experimented upon, and upon the pressure on the after portion exceeding that of the front portion the pin was released and fell down. These experiments proved the very important fact that from the vertical to an angle of 45° the centre of pressure was at the centre of the plane; but that from 45° to 10° the centre of pressure shifted forward to as much as 15 per cent. of the whole plane—at 30°, 20°, 15° and 10° the centre of pressure became uniformly 35 per cent. from the front edge, and 65 per cent. from the after edge.

"I fully expected that varying speeds would give varying results as to this, and was suprised that I could not obtain them; I hope, however, soon to resume the experiments in order to obtain more light upon this interesting and important point, and also to prove the results obtained by Mr. Browning's anemometer and to extend the experiments to finer angles than were tried at Messrs. Penn's establishment.

"In conclusion I may add that flexible planes will form an important series in the experiments to be tried."

The actual report, in Moy's handwriting, is filed in the Minute Book of the Council of that year.

Of course these experiments were crude compared with present-day methods, and were not as accurate. But it took a major war to get better answers, and, full circle, war has made demands which have left members of the Society urging more and more should be spent on aerodynamic research, for what Moy failed to find with £15 with planes moving at low speeds, present wind tunnels have not succeeded in finding with £15,000,000 at high speeds. Eighty years ago what members of the Society were lacking in data and information on aerodynamics they made up for in enthusiasm and *ad hoc* attempts with flying models. Now too much power has overshadowed the lack of information of aerodynamic problems at high speeds, and supersonics have made us deaf to the advantages of the slow but sure method of progress.

Let me finish with a quotation from that great American member of the Society, Octave Chanute, the friend of the Wright Brothers, writing in *The Railroad and Engineering Journal* of New York for October 1891.

"I had a chart plotted on which were delineated all the experiments on inclined surfaces which I could learn about—those of Hutton, Vince, Thibault, Duchemin, de Louvrie, Skye, the British Aeronautical Society and W. H. Dines; and on this chart I also had plotted the curves of the various formulas. The whole exhibited great discrepancies, yet by patient analysis various probable sources of error were eliminated, and the conclusion was reached that the formula last given, which I have seen variously attributed to Bossut or to Duchemin, was probably correct."

That formula, for those who do not know it, is

$$P = P' \frac{2 \sin z}{1 + \sin^2 z}$$

where P is the pressure on the inclined surface, P' the pressure on the surface when at 90° to the air flow, and z is the angle the inclined plane makes with the current of air. But of all these investigations, up to the time Chanute wrote in 1891, W. H. Dines, in my opinion, showed the best experimental methods and results. Past that date I do not feel inclined to go for the moment, for the very air above our heads has become secret and to talk about it may lead to a turbulence of the soul beyond our control. No longer can we say with our national poet (habit almost made me say nationalised poet):

"The isle is full of noises,
Sounds and sweet airs, that give delight and hurt not."

Stress Analysis of Circular Frames in a Non-tapering Fuselage

by

K. J. DALLISON

(Stress Office, Handley Page Ltd.)

SUMMARY:—The problem of the stress analysis of circular fuselage frames has been investigated by a number of authors; the analyses are, necessarily, rather mathematical in nature although in some cases the final results have been presented in extremely practical form. The present paper offers, in non-mathematical language, a short account of the fundamentals of the problem and a brief guide to the more important published literature. It also develops an approximate method for the analysis of frames using Lagrange's method for minimising a function of several inter-dependent variables. This method is shown to be particularly suitable for the analysis of frames with large cut-outs, about which little has previously been published. In addition, the underlying parameters which determine the stress distribution are deduced from the theory, and the deductions are compared with previously published work. In general, agreement is good. The paper is written throughout with a view to being of practical use in the actual stressing of frames. Numerical examples and explanations of mathematical methods are included in Appendices.

1. Introduction

The paper falls naturally into three parts:—

I. A review of the whole field of currently accepted frame theory, together with brief notes on some of the more important of published investigations.

II. The development of an original method of analysis, using Lagrange's undetermined multipliers for minimising strain energy, the method being particularly adapted to the solution of frames with a large cut-out.

III. A discussion of the underlying parameters which determine the stress distribution, using the results of previously published work and predictions from the theory developed in Part II.

Notation

$*A = r^6\ t'/(L^3\ I)$ (or $6\ r^6\ t'/(L^3\ I)$ for Ref. 4. *See* Section 19)

$*A$ (Ref. 5 only) area of cross section of frame. *See* Section 20

$*B = E\ t'\ r^2/(G\ t\ L^2)$ (or $6\ E\ t'\ r^2/(G\ t\ L^2)$ for Ref. 4. *See* Section 19)

$*A/B = Gtr^4/(EIL)$

$C = I/(S\ r^2)$

$D = ES/(GS')$

a_{mn} product of l and "offset from centre of m^{th} section to tangent at skin line of n^{th} section." *See* Section 14.

c_m, d_m offsets defined and used in Appendix V

c, d, e parameters used in Refs. 1, 8 and referred to in Section 19

E Young's modulus

G shear modulus

g number defining section of frame

I, I_{frame}, I_{beam} moments of inertia of cross sections of frame and floor beam

I_m mean value of moment of inertia of cross section of frame over m^{th} strip

K, K_3 constants

$K_1 = L\ l/(2tG)$

$K_2 = l/(2E)$

L assumed length of fuselage in which distorted shear reverts to basic form (*see* Section 14(*i*))

L (Section 19 only) average frame spacing

l length of one section, measured in skin line

M bending moment

M_a applied couple

M_1 free bending moment in floor beam (Appendix II)

M_2 redundant moment (Appendix I)

M_m bending moment at centre of m^{th} section of frame due to shear in skin in sections 1 to m

M_m' bending moment at centre of m^{th} section due to external applied forces P, T or M_a

m number defining section of frame

$m_1, m_2, \ldots m_6$ coefficients used in Appendix II

N internal redundancy (axial load) in Appendix I

n number defining section of frame

P applied radial load

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*For A, A/B and B, L is to be taken as mean frame spacing.

p number defining section of frame
Q_n defined and used in Appendix III
q shear per unit length, or "shear flow" in skin
R radius of skin centre line in Ref. 1, 5, 8 (*see* Section 19)
r radius of skin centre line
r constant used in Ref. 8 (*see* Section 19)
r' radius of centre line of frame, if different from r
S cross-sectional area of frame (*i.e.* end load carrying area)
S' cross-sectional area of frame web (*i.e.* shear carrying area)
t skin thickness
t' thickness of equivalent end load carrying sheet replacing skin and stringers
U strain energy
V internal redundancy used in Appendix I
$2W$ distributed floor load in Appendix V
X, X_1, X_2 internal redundancies used in Appendices II, V
X_1, X_2 (Section 17 only) restraining forces from wing
$x_1, x_2, x_3, \ldots, x_p, \ldots x_n$ shear per unit length or "shear flow" in skin at first, second,... p^{th}, ... n^{th} sections of frame
x distance along floor beam (Appendices II, V)
$\alpha, \alpha_m, \alpha_n, \beta, \gamma, \theta, \phi, \delta_1, \delta_2$ angles defining points of frame
λ, μ undetermined multipliers for use with Lagrange's method of minimising a function of several interdependent variables (*see* Appendix VI)
η eccentricity $=(r-r')/r$
ν Poisson's ratio
$\lambda_A =$ (maximum bending moment, shear flow, etc., for a given value of A)/(maximum bending moment, shear flow, etc., for $A > 10^6$)
$\lambda_\eta =$ (maximum bending moment, shear flow, etc., for a given value of η)/(maximum bending moment, shear flow, etc., for $\eta = 0$)
ξ used and defined in Appendix VII.

PART I

A REVIEW OF CURRENTLY ACCEPTED FRAME THEORY

2. Statement of the Problem

When one of the circular frames of a cantilevered monocoque fuselage of constant diameter is loaded (*see* Fig. 1), the load is reacted by shear in the skin, together with associated end loads. The reacting shear gives rise to stresses in the frame, *i.e.* bending moment, axial load and web shear. For a complete stress analysis all of these must be determined.

The solution falls naturally into two parts: (*a*) the determination of the distribution of the reacting shear in the skin; (*b*) the determination of the bending moment, et cetera, in the frame. If the skin shear distribution is known, (*b*) consists fundamentally of the evaluation of two redundancies (*see* Section 9). If the skin shear distribution is not known, the values of these redundancies are generally obtained as a by-product from the solution of (*a*). (*b*) is thus a relatively simple problem, but (*a*) is in general highly complex. The determination of the skin shear distribution constitutes the crux of the problem.

3. Types of Circular Frames

Circular fuselage frames can be (arbitrarily) classified into five types, all of which are of common occurrence in practical aeronautics:—

(*i*) Complete frames of constant cross section, with no internal reinforcing.
(*ii*) As (*i*) but with internal reinforcing in the form of floor beams and supporting struts.
(*iii*) Frames with a cut-out.
(*iv*) Frames with a variable cross section.
(*v*) Complete or partial bulkheads.

Unless otherwise stated, Sections 5 to 10 refer to type (*i*). Types (*ii*) to (*v*) are dealt with in Sections 11 onwards.

4. Types of Frame Loading

The following types of loading are considered:—

(*i*) Radial loads applied direct to the frame.
(*ii*) Tangential loads applied direct to the frame.
(*iii*) Moment loads applied direct to the frame.
(*iv*) Loads applied to the frame through a floor beam.

Other systems of loading can be treated as a combination of these four.

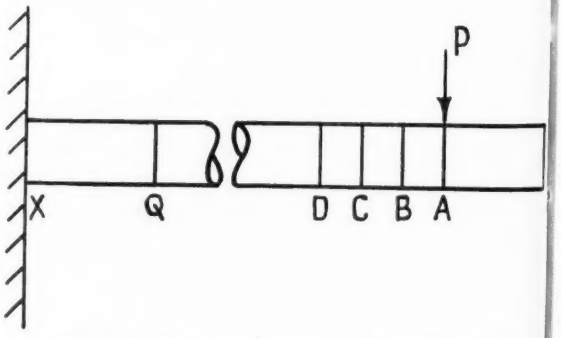

FIGURE 1. Cantilevered monocoque fuselage loaded at frame A.

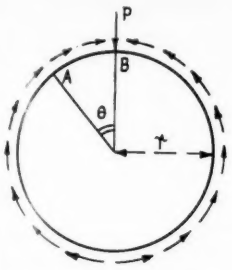

FIGURE 2. Radially loaded frame showing skin shear reactions.

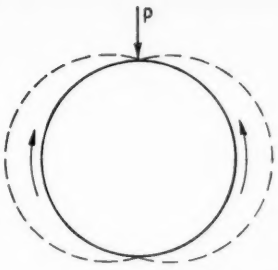

FIGURE 3. Skin shear distribution due to radial load (rigid frame).

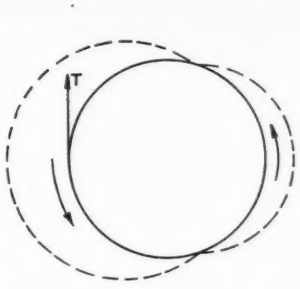

FIGURE 4. Skin shear distribution due to tangential load (rigid frame).

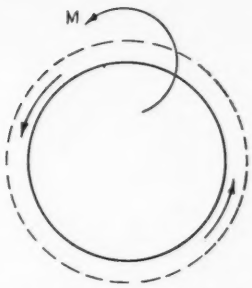

FIGURE 5. Skin shear distribution due to moment load (rigid frame).

5. Distribution of the Reacting Shear

When the applied load is radial, simple engineering bending theory requires that the reacting shear should be distributed sinusoidally; *i.e.* at a point A in Fig. 2 defined by an angle θ measured from the diameter through the point of application of the load, the shear per unit length or "shear flow" q is given by $q = K \sin\theta$.

The value of K is readily obtained by considering overall balance in a direction parallel to the applied load.

Hence $$2\int_0^{\pi} K \sin^2\theta \, r \, d\theta = P$$

yielding $$K = \frac{P}{\pi r} \text{ and } q = \frac{P}{\pi r}\sin\theta \quad . \quad . \quad (1)$$

If a tangential load T or a moment load M is applied at B (Fig. 2), simple engineering theory similarly gives the shear flow at A as

$$q = \frac{T}{\pi r}(\tfrac{1}{2} + \cos\theta) \quad . \quad . \quad (2)$$

and $$q = \frac{M}{2\pi r^2} \quad . \quad . \quad . \quad . \quad (3)$$

respectively. These shear distributions are illustrated diagrammatically in Figs. 3, 4 and 5.

Engineering experience suggests that in the neighbourhood of the loaded frame there may be some distortion of the pattern of the skin shear distribution. Both experiment and calculation confirm this, if the loaded frame is flexible. Moreover the distortion of the shear pattern gives relief to the bending stresses in the frame. If, however, the frame is rigid there is no distortion of the shear pattern.

The terms "rigid" and "flexible" are comparative, not absolute, and require definition. A suggested definition will be found in Section 14(*j*).

When there is variation of skin thickness round the periphery of the loaded rigid frame, the skin shear distribution given by equations (1)-(3) will be modified. This modified shear can still be calculated by engineering theory (the shear stress being given by some expression such as $FS\bar{y}/(IB)$ in the notation of Case, *Strength of Materials*, p. 202). But it is still the "basic" shear. Distortion of the skin shear pattern due to this cause must not be confused with distortion due to frame flexibility.

6. Nature and Magnitude of the Deformation of the Skin Shear Distribution Pattern (and Bending Moment Pattern) at a Flexible Loaded Frame.

Figures 6 and 7 show typical skin shear and frame bending moment distributions for a flexible frame, loaded radially, compared with similar distributions for a rigid frame. The figures are diagrammatic only and represent the type of result obtained by various investigators. Actual curves can be seen in the quoted references.

The ratio q_1/q_2 in Fig. 6 gives a measure of the distortion of the shear pattern. Hoff[3, 4] calls it the "shear concentration factor" and examines its variation with parameters representative of frame and skin stiffnesses. Hoff also plots the "moment reduction factor" (represented by m_1/m_2 in Fig. 7) against the same parameters. Similar results are obtained with tangential and moment loadings.

For the most flexible frame examined Hoff found shear concentrations as high as 35, and moment reduction factors as low as 0·33 (*see* Ref. 4, Figs. 5 and 6). Values outside these limits could occur with still more flexible frames, but practical considerations will put a limit to the flexibility of frames actually used in aircraft structures.

7. Reversion of the Distorted Shear to Basic Form

If a flexible frame A well away from X, the supported end of the cantilever (*see* Fig. 1), is loaded radially, the skin shear distribution in its immediate neighbourhood will be distorted in the manner of Section 6. But at X the shear will have taken up its basic form again. The change occurs in steps at the intervening frames $B, C, D, E, \ldots\ldots$ The number of frames required to "revert" the distorted shear pattern to basic depends on their flexibility. One fairly rigid frame might suffice. With very flexible frames, as many as six might be required. (*See* Ref. 8, Fig. 7; Ref. 11, Fig. 4; Ref. 2, Fig. 4.)

Stresses will be set up in the succeeding frames B, C, D, and so on, depending on the difference in the

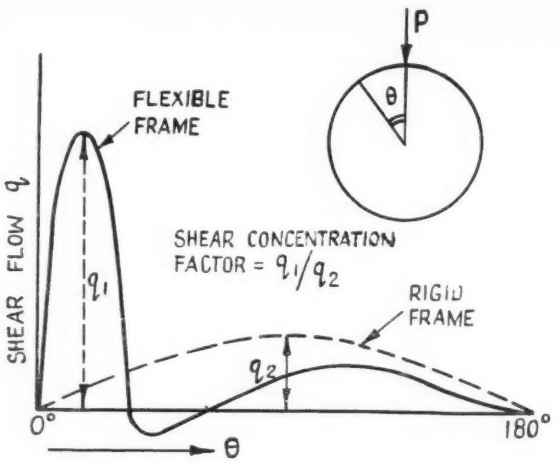

FIGURE 6. Typical skin shear patterns for flexible and rigid frame.

shear distribution on the two sides of such a frame. These stresses are plotted for various cases in Refs. 2 and 7. In general they are small, but not necessarily negligible.

If no intermediate frames occurred, there would still be a tendency for the shear to revert to basic before X, each ring of skin acting as a very flexible frame. There *can*, therefore, be a change of shear pattern without intermediate frames. But when frames are present, and not too widely spaced, strain energy considerations demand that this change shall occur chiefly at the frames. Hence in current frame theory, the shear distribution is treated as constant between two successive frames.

If a frame is loaded with tangential or moment loading, similar changes in the shear pattern occur.

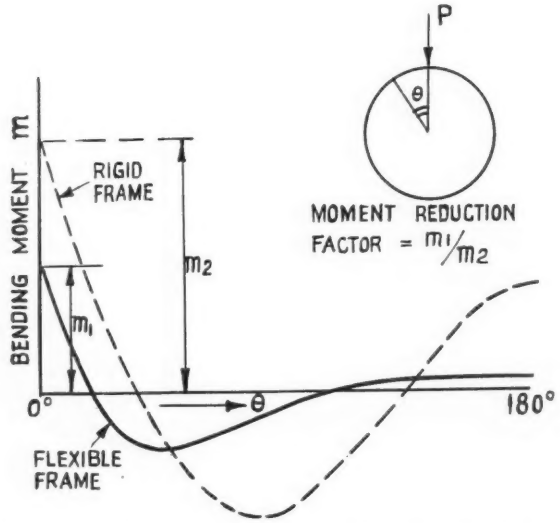

FIGURE 7. Typical bending moment distributions for flexible and rigid frames.

8. Factors that Determine the Shear Distribution

These are:—

(*i*) The second moment of area of the frame cross section (I),

(*ii*) Fuselage skin thickness (t),

(*iii*) Radius of fuselage (r),

(*iv*) Frame spacing,

(*v*) Eccentricity η, *i.e.* offset of centre line of skin from centre line of frame, expressed as a fraction of r, the skin radius,

(*vi*) Stringer area and spacing and, to a less extent, bending and torsional stiffness of stringers,

(*vii*) The number of "fields" between the loaded frame and the point of support,

(*viii*) The number of "fields" between the loaded frame and the free end of the fuselage,

(*ix*) The extensional and shearing rigidities of the frame.

The term "field" is to be taken as indicating the portion of fuselage between two successive frames.

The relative importance of these factors is discussed in Part III.

9. Determination of the Bending Moment in a Complete Frame of Constant Cross Section with a Given Distribution of the Reacting Shear

Figure 8 shows a frame loaded radially at a point A. An imaginary cut is made at a diametrically opposite point B, and internal forces N (axial load), V (shear) and M_2 (moment) are applied to the cut edges.

If the frame is symmetrical about AB, the shear distribution will also be symmetrical. Then, from symmetry, V is zero, and the problem resolves itself into the determination of two internal forces only, N and M_2. These constitute the redundancies of the system.

If a tangential load T or a moment load M_0 is applied to the frame A, the redundancies again reduce to two in symmetric frames. (For T, make the cut at 0°. N is $P/2$ and $-P/2$. Redundancies are M_2 and V. For M_0 make the cut at 0°. M_2 is $M_0/2$ and $-M_0/2$. The redundancies are N and V.)

One of the first published solutions to this problem was due to Wise[9]. Assuming the distribution of skin shear given by simple engineering theory (*see* equations (1) to (3) of the present paper), he evaluated the redundancies for a radial, tangential or moment load, and hence calculated the bending moment, axial load and web shear at any point in the frame. His results were published in the Royal Aeronautical Society's Stressed Skin Data Sheets (03.06 series), but since he used a shear distribution based on engineering theory, his results apply to rigid frames only.

Wise obtained his solution by using a "method of consistent deflections." The problem is equally amenable to routine strain energy analysis, and a solution by this method is outlined in Appendix I.

If the original shear distribution is given in graphical, not analytic, form, the strain energy method of Appendix I still applies, but the integrations must be performed graphically. This case could arise when the skin in the neighbourhood of a rigid frame is varying in thickness round the frame, the shear being determined by engineering theory but not being capable of analytic expression (*see* Section 5). In other cases, as already stated, determination of the shear pattern generally yields the values of the redundancies and further strain energy analysis is unnecessary.

Morse[13] suggests that the calculation of the bending moment from a known shear distribution is facilitated by applying the redundancies at the "elastic centre," which he defines in terms of the elastic properties of the frame. This seems to simplify the calculation in certain cases.

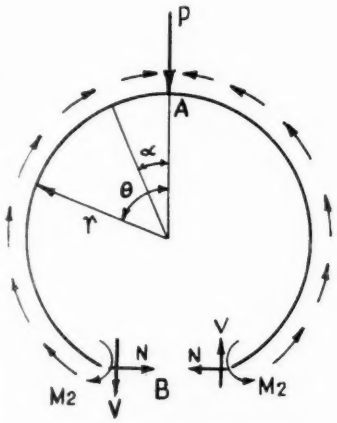

FIGURE 8. Radially loaded frame, cut to show redundancies.

10. Determination of the Skin Shear Distribution at a Loaded Flexible Frame

The determination of the skin shear distribution at a loaded flexible frame is a matter of considerable difficulty. It has been attacked by several investigators from varying points of view, but the method most favoured has been to represent the shear distribution in the skin by a trigonometric series (*e.g.* the shear flow at a point defined by θ is taken as $a_0 + a_1 \sin\theta + a_2 \sin 2\theta + a_3 \sin 3\theta + \ldots + b_0 + b_1 \cos\theta + b_2 \cos 2\theta \ldots$). The strain energy of the loaded frame and adjacent skin is then written down in terms of the assumed shear flow, and minimised with respect to each of the coefficients $a_0, a_1, a_2 \ldots b_0, b_1, b_2 \ldots$ and so on, in turn. Alternatively differential equations are set up which ensure compatibility between the skin and the frame, and these are solved by any convenient artifice.

Analyses of this type are long and cumbersome but some of the investigators have presented their results in an eminently practical form. Others have not done so and it is difficult to make practical use of their work.

Brief comments on some of the more important of the published analyses are appended.

REFERENCE 1

This was the first published report showing the effect of fuselage frame flexibility. By ensuring compatibility between skin and frame deflections the authors established a differential equation whose solution gave them the shear flow. No allowance was made for the effect of eccentricity or stringer flexibility under end load (*see* Sections 19 and 20 of the present paper). The report contains a discussion of underlying physical concepts, and nests of curves similar to those discribed in Ref. 7 are presented. Unfortunately, these will be substantially in error in many practical cases, *i.e.* for low values of A (*see* Section 19 and Fig. 29 of the present paper).

REFERENCES 2, 3 AND 4

Hoff and his associates analysed frames using strain energy, the shear flow being represented by a Fourier series and the strain energy minimised with respect to the successive coefficients. Ref. 3 is an advance on Ref. 4 in that more of the factors affecting shear distribution are included in the analysis, and allowance is made for radial reactions between the skin and the frame. Ref. 4 contains curves showing the effect of relevant factors on skin shear and frame bending moment. These are referred to again in Sections 19-21.

Hoff deals only with radial loading and gives only frame bending moment and skin shear. His curves therefore do not have the same completeness as those in Ref. 7. Skin shear and frame bending moment can be calculated for tangential and moment loadings from the curves for the radial loading by using Goodey's method (*see* Section 16 of the present paper), but axial load and web shear in the frame can only be deduced after the evaluation of two redundancies, as explained in Section 9. As the values of these redundancies must have been obtained at some time during the calculation, the practical utility of these curves would have been enhanced if their values had been plotted or tabulated.

REFERENCE 5

Beskin used a strain energy method, representing the skin shear flow by a Fourier series. He included the effects of frame eccentricity and also of shearing and extensional rigidities of the frame. He produced curves for frame bending moment, frame axial load and frame web shear for radial, tangential and moment loads, with parametric variation with skin eccentricity. These curves give the best available picture of the effect of eccentricity on frame stress distribution. They are discussed more fully in Section 20 and a cross plot is presented in Fig. 30.

REFERENCES 6 AND 7

Duberg and Kempner[6] used self-equilibrating systems of stress which they added to the basic stress system to give compatible displacements, relating these self-equilibrating systems in successive bays by recurrence formulae. They did not allow for eccentricity. Hoff, in Ref. 3, claims that their solution agrees with his earlier solution.

Reference 7 contains families of curves based on Ref. 6. These form the readiest available source of information on stresses in flexible frames. The skin shear and end load, the frame bending moment, axial

load and web shear are plotted round the frame for radial, tangential and moment loads with parametric variation against two factors representative of frame and stringer stiffnesses. There are some 45 pages of these nests of curves. The loading of the adjacent frame is also shown. The curves require correction for the effect of skin eccentricity (*see* Fig. 30).

REFERENCE 8

Goodey's analysis is perhaps the most fundamental of the published analyses. He starts by considering the effects of all the factors that control the skin shear distribution, including all those listed in Section 8 of this paper and also including the Poisson effect and the normal reaction between skin and frame. Later, in the interests of simplicity, he drops a number of these; notably he ignores stringer bending and torsion and uses zero eccentricity.

Despite the fundamental nature of his analysis, his results have not been of great practical use owing to the complex mathematics and the paucity of published curves. Ref. 11 is a welcome step towards making his results more readily applicable.

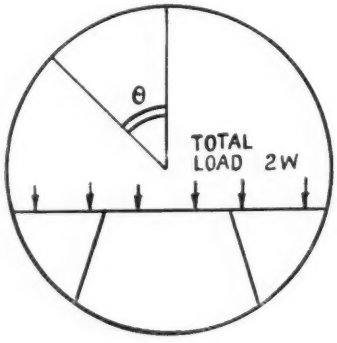

FIGURE 9. Typical circular fuselage frame with propped floor beam carrying distributed load $2W$.

REFERENCE 11

This report sets out to give curves based on Ref. 8 which can be used for the practical stress analysis of frames. The curves apply only to radial loading, but Goodey's method for deducing the corresponding values for tangential and moment loading (*see* Section 15) is described in detail.

The curves give skin shear only. To determine the bending moment, et cetera, in the frame, two redundancies must be evaluated, and this necessitates strain energy analysis using the given shear distribution. As explained under Ref. 4, the value of the report would have been enhanced if these redundancies had been tabulated. The curves also require correction for eccentricity.

Curves showing the variation of the skin shear with I and t are also presented. These are discussed in Section 19.

11. Frames of Constant Section with Internal Reinforcing

Figure 9 shows a typical frame of constant section with a propped floor beam, loaded with a distributed floor load $2W$. There will be inter-reactions between the frame and the beam, and these are shown in Figs. 10 and 11.

Using a set of curves such as the Royal Aeronautical Society Stressed Skin Data Sheets, 03.06 series, for a rigid frame or Ref. 7 for a flexible frame, the stress analysis is comparatively simple, being essentially the determination of three redundancies, M_0, P and X. The

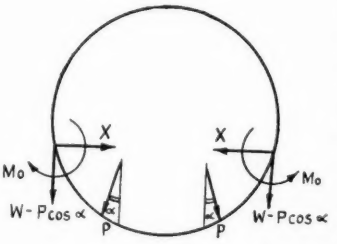

FIGURE 10. Frame of Fig. 9 showing forces applied to frame by floor beam and props.

problem can be solved by conventional strain energy methods, and one such solution is given in Appendix II. It contains nothing new, but is includeed for completeness and easy reference. A solution of a similar problem for a rigid frame (*i.e.* using skin shear from basic engineering theory) was published in Ref. 10.

12. Cut Frames and Frames with Varying Section. General Remarks

Fuselage frames with large cut-outs are of common occurrence in aircraft, *e.g.* at hatches, in doorways and in bomb bays, but hardly any published work touches on the problem of their stress analysis. Ref. 12, for instance, gives the effect on the shear in the fuselage of a torsional loading remote from a long cut-out. But it does not examine the part played by the frames themselves in the redistribution of this shear, nor does it deal with a load applied *at* a frame with a cut-out, which is the problem under discussion here.

Similarly, little work has been published on frames of widely varying section, although Lewis[11] has made some attempt to assess the significance of changes in I.

As a first approach, some attempt might be made to modify published solutions for complete frames and to apply them to frames with cut-outs. Most of these solutions, however, use trigonometric series, which are relatively easy to handle between limits such as 0° and

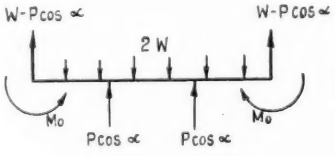

FIGURE 11. Forces applied to floor beam by frame and props. (Horizontal components omitted.)

$\frac{1}{2}\pi$ or 0° and π, but are much more intractable over intermediate ranges such as 0° and 130°.

Some other artifice must therefore be adopted. The author has developed a simple *ab initio* method of frame analysis which is particularly suitable for frames with large cut-outs. It can be applied with sweeping idealisations to give a quick solution for design. It can be extended to include greater detail, although it soon

becomes unwieldy. It yields some interesting theoretical results.

The method is described in Part II. No attempt is made to present the results in the form of comprehensive nests of curves, as the number of curves required would be prohibitive. A rapid method of calculation for a given case seems the best that can be hoped for at present.

PART II

ANALYSIS OF FUSELAGE FRAMES BY THE STRIP METHOD

13. Introduction

The method consists essentially of dividing the periphery of the skin into a number of equal strips, denoting the average shear flow on each of these strips by x_1, x_2, x_3 . . . x_p, and minimising strain energy by Lagrange's method.

As the shear in an actual fuselage is usually applied to the frame in discrete steps, at first sight the strip method would appear to give a closer approximation to reality than analyses which represent the shear flow by continuous functions. To keep the number of variables down to reasonable limits, however, it is generally necessary to make the number of steps in the calculation less than the actual number of steps that occur in practice, so this advantage is unfortunately lost.

A detailed solution for a frame with a cut-out, loaded radially, is set out in Section 14, and the method of solution for tangential and moment loadings is indicated. Numerical examples are worked in Appendices III and IV.

14. Analysis of a Frame with a Cut-out Loaded Radially at a Point Diametrically Opposite to the Centre of the Cut-out

Figure 13 shows a frame with a cut-out, loaded with a radial load P at a point diametrically opposite to the centre of the cut-out. Let the half frame AB be divided into p sections of equal length l. Let the average values of the shear flow in the skin over the n^{th} strip be x_n, and the angle to the centre of this strip be α_n; x_1 and α_1 refer to the strip nearest to the cut-out (*see* Fig. 13).

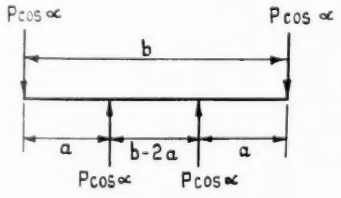

FIGURE 12. Effect of P on bending moment of floor beam (*See* Appendix II).

The bending moment at the centre of the m^{th} strip due to shear on the n^{th} strip ($n \geqslant m$) will be $a_{mn}x_n$ where a_{mn} is the product of the length of the skin in one section and the geometrical offset from the centre of the m^{th} section of the frame to the line of action of the shear force on the n^{th} section.

Let the total bending moment at the centre of the m^{th} section be M_m. Then

$$M_m = \sum_{n=1}^{n=m} a_{mn}x_n \quad . \quad . \quad . \quad (4)$$

also $$\frac{\partial M_m}{\partial x_n} = a_{mn}(n \geqslant m), \quad \frac{\partial M_m}{\partial x_n} = 0 \ (n > m) \qquad (5)$$

The strain energy of the half frame in bending will be

$$\frac{l}{2E} \sum_{m=1}^{m=p} \frac{M_m^2}{I_m}$$

where I_m is the average value of I for the m^{th} section.

Assume now that the skin shear distribution persists unchanged for a length L along the fuselage (*see* (*i*) of this section), and then reverts to basic sinusoidal form. The strain energy in shear of the skin in half the fuselage over the length L will be

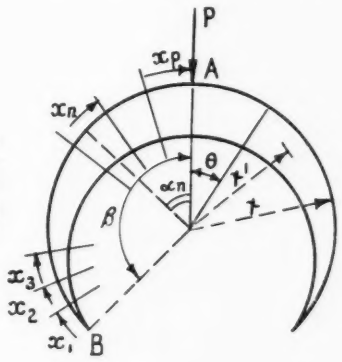

FIGURE 13. Cut frame (varying section) divided into p segments each loaded with a discrete increment of shear (Section 14).

$$\frac{Ll}{2tG} \sum_{n=1}^{n=p} x_n^2.$$

The total strain energy of one half of the frame and fuselage skin is thus given by

$$U = K_1 \sum_{n=1}^{n=p} x_n^2 + K_2 \sum_{m=1}^{m=p} \frac{M_m^2}{I_m} + k_3 \quad . \qquad (6)$$

where K_1, K_2, K_3 are terms independent of x_1, x_2 . . . x_n: K_1 being $Ll/(2tG)$, K_2 being $l/(2E)$ and K_3 representing the strain energy in shear of the remainder of the half fuselage beyond the length L. (This ignores the energy of the stringers due to end load. This is considered in (*g*) of this section.)

Another relation follows from consideration of the static equilibrium in a direction parallel to the line of application of P,

i.e. $$\frac{P}{2l} = \sum_{n=1}^{n=p} x_n \sin \alpha_n \quad . \quad . \quad . \quad . \quad (7)$$

From (6) and (7), minimising by Lagrange's Method of Undetermined Multipliers (*see* Appendix 6):—

$$\lambda \sin \alpha_n = 2K_1 x_n + 2K_2 \sum_{m=1}^{m=p} \frac{M_m}{I_m} a_{mn} \quad (m \lessdot n) \qquad (8)$$

i.e.

$$\lambda \sin \alpha_n = 2K_1 x_n + 2K_2 \sum_{m=n}^{m=p} \frac{M_m}{I_m} a_{mn} \quad . \qquad (9)$$

From consideration of this result the following points arise:—

(*a*) If I_m is varying round the frame, it remains inside the summation sign, but if it is constant it can be taken outside. Similarly if t is varying round the perimeter, it must be removed from K_1 and placed inside the summation sign for the shear term.

(*b*) When $I_m \longrightarrow \infty$ (rigid frame), from (9),

$$\lambda \sin \alpha_n = 2K_1 x_n, \quad i.e. \ x_n \propto \sin \alpha_n,$$

i.e. the shear distribution is sinusoidal. Thus for a rigid cut frame loaded as in Fig. 13, the shear at any

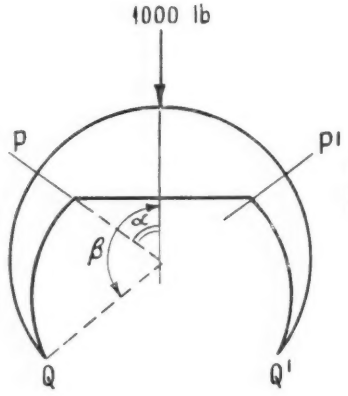

FIGURE 14. Typical partial bulkhead loaded radially at top centre line.

point α may be written as $K \sin \alpha$ where K is given by

$$2r \int_0^{\beta} K \sin^2 \alpha d\alpha = P, \quad i.e. \ K = \frac{P}{r(\beta - \frac{1}{2} \sin 2\beta)} .$$

(*c*) For a complete rigid frame, the shear distribution will also be sinusoidal, since the redundancies M_2 and N of Section 9 give terms in the bending part of equation (8), and these vanish when $I_m \longrightarrow \infty$. Moreover, putting $\beta = \pi$ in the expression for K, the shear flow at point α becomes $[P/(\pi r)] \sin \alpha$ which is in complete agreement with simple engineering theory (Section 5).

Values of the shear flow for complete or partial rigid frames loaded with tangential or moment loadings can similarly be obtained, and are also found to agree with simple engineering theory (*see* Section 15(*e*)).

(*d*) When the frame is rigid over a considerable portion of its perimeter, this portion being symmetrically disposed about A (Fig. 13), and extending from $n = g+1$ to $n = p$ (say), then from (9),

$$n \geqslant g, \quad \lambda \sin \alpha_n = 2K_1 x_n + 2K_2 \sum_{m=n}^{m=g} \frac{M_m a_{mn}}{I_m}$$

$$n > g, \quad \lambda \sin \alpha_n = 2K_1 x_n,$$

i.e. over the rigid portion of the frame, shear flow is sinusoidal. This result is of great practical value, as the shear flow from $n = g+1$ to $n = p$ can now be represented by $K \sin \alpha$ where K is the only unknown. This simplifies the solution (but *see* Appendix VII).

This method is thus particularly suitable for the solution of partial bulkheads of the type shown in Fig. 14. A numerical example of such a solution is worked in Appendix III. In the example given there are seven equations to solve, five of which can be solved separately from the other two. Thus the solution is obtained without excessive work.

(*e*) When I_m is finite, p equations of the form (9) are obtained, giving with equation (7) a total of $p+1$ equations. There are $p+1$ unknowns, namely x_1 to x_p and Lagrange's multiplier λ. The solution is thus determinate.

(*f*) For a more accurate solution the procedure is as follows. The frame is solved with a small number of strips as in Appendix III. Then a new block of equations is written down, increasing the number of strips as required. From a "faired-in" graph of the first solution, approximate solutions of these equations may be read off and more accurate values can be obtained quite speedily by relaxation or by applying Von Seidel's method (*see* Appendix VI).

(*g*) For very flexible frames, the shear flow obtained by this method shows a tendency to oscillate between very large positive and negative values at successive stations, giving a quite unrealistic solution. The explanation is that rapidly fluctuating shear of this kind would be associated with high end load in the skin and stringers. The strain energy due to this end load has been ignored. If it is included, the peaks of the shear curve will be planed off. Computational details are given in Appendix V, and an indication as to when this energy of skin and stringers in end load should be included in the calculation is given in Section 19. (The validity of ignoring stringer energy at a rigid frame is also discussed).

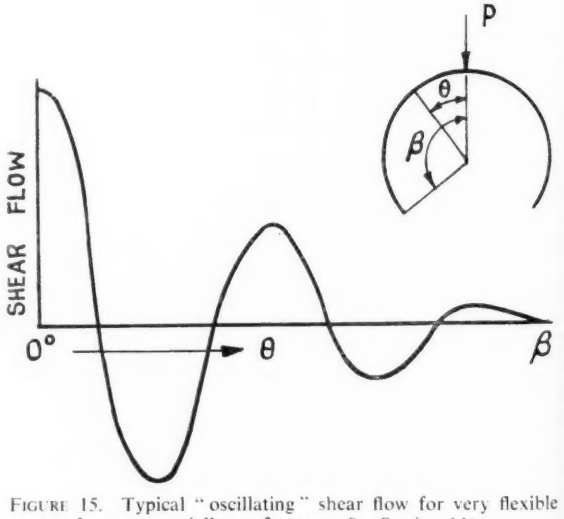

FIGURE 15. Typical "oscillating" shear flow for very flexible frames (especially cut frames). *See* Section 14(*g*).

The tendency for the shear to oscillate still persists for a very flexible frame, and this is borne out by the curves in Ref. 7 for high values of the parameters A and A/B (*see* also Section 19).

(*h*) The effect of eccentricity has been allowed for automatically in this method of analysis by the use of a_{mn} (*see* Sections 8 and 20).

(*i*) The choice of a length L in which the shear reverts to basic presents some difficulty. If there is a fairly massive frame only one or two fields away from the loaded frame, L should be taken as the distance to this frame. If succeeding frames are flexible, L should not exceed three times the average frame spacing (*i.e.* effectively allowing the reversion to take place in steps over six fields). Unless the bending and shear terms in equation (9) happen to come fairly equal in magnitude, however, the final result will not be very sensitive to the assumed value of L (*see* Appendix III for numerical examples).

Alternatively, instead of choosing an arbitrary value for L, shear flows in succeeding fields could be denoted by $y_1, y_2, y_3 \ldots y_p$ and $z_1, z_2, z_3 \ldots z_n$, and the strain energy of the skin in shear and the frames in bending in these fields could be written down and added to the original energy. The work will be greatly increased, as the original equations will now contain terms in x and y, and there will be an additional set of equations with terms in y and z. The final solution will be more accurate, but the method has lost its chief virtue, namely its simplicity.

(*j*) It is now possible to define a "rigid" frame and a "flexible" frame. A rigid frame is a frame whose strain energy in bending is small compared with the strain energy in shear of the skin in adjacent bays. Conversely, a flexible frame is one in which this is not the case. This definition agrees with Ref. 1, page 15. It is given a quantitative character in Section 19 of the present paper.

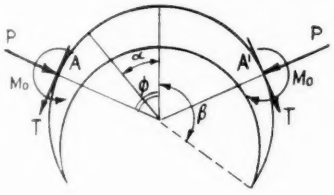

FIGURE 16. Symmetrically placed radial, tangential and moment loadings.

(*k*) It must be pointed out that when partial or complete bulkheads are examined by the methods of this paper, no information is given on the distribution of the shear stresses in the bulkhead web. This could be determined by ordinary engineering methods, or by more refined analysis, but in either case the skin shear distribution given by this paper could be used as a starting point, since it is unlikely that the web energy would be high enough materially to affect the skin shear distribution.

(*l*) As in all strain energy calculations, the main block of equations will form an array symmetrical about the leading diagonal. Special methods exist for the solution of such equations. For the convenience of readers, some of these methods are outlined in Appendix VI.

(*m*) It might be argued that a substitution method could be used in preference to Lagrange's undetermined multipliers. This is true, but the elegance and symmetry are lost, and the theoretical results emerge less clearly.

15. Cut Frames with Other Forms of Loading and with Internal Restraints

(*a*) *Two symmetrically placed radial loads*

If the radial loads P are applied at A, A' (Fig. 16), the bending moment M_m at all sections where $\alpha_m < \phi$ will contain a term in P. Equation (4) will be modified to:—

$$\left.\begin{aligned} \alpha_m < \phi, \quad & M_m = \sum_{n=1}^{n=m} a_{mn}x_n - Pr'\sin(\phi - \alpha_m) \\ \alpha_m \not< \phi, \quad & M_m = \sum_{n=1}^{n=m} a_{mn}x_n \end{aligned}\right\} \quad . \quad (10)$$

where r' is radius of centre line of frame at section m. The equilibrium equation now becomes

$$\frac{P}{l}\cos\phi = \sum_{n=1}^{n=p} x_n \sin\alpha_n \quad . \quad . \quad (11)$$

Equation (9) is unchanged.

Reference to Appendix III shows that the bulk of the computational work from the previous (radial) loading stands unchanged. The constants on the right hand side of the block of equations must be computed afresh and equation (11), giving the value of λ, is different. These are the only alterations. The solution to this loading can thus be obtained expeditiously from the solution to the preceding loading.

(*b*) *Two symmetrically placed tangential loads* (*see* Fig. 16)

At sections where $\alpha_m \not> \phi$ the expression for M_m will contain a term involving T. Equation (4) will be modified to

$$\left.\begin{aligned} \alpha_m \not> \phi, \quad & M_m = \sum_{n=1}^{n=m} a_{mn}x_n - T[r - r'\cos(\phi - \alpha_m)] \\ \alpha_m > \phi, \quad & M_m = \sum_{n=1}^{n=m} a_{mn}x_n \end{aligned}\right\} \quad (12)$$

The equilibrium equation becomes

$$\frac{T}{l}\sin\phi = \sum_{n=1}^{n=p} x_n \sin\alpha_n \quad . \quad . \quad (13)$$

Equation (9) is unchanged. Also, as in (*a*), the solution follows readily from the numerical solution of the case dealt with in Section 14.

(*c*) *Two symmetrically placed moment loads* (*see* Fig. 16)

Equation (4) becomes

$$\left.\begin{aligned} \alpha_m < \phi, \quad & M_m = \sum_{n=1}^{n=m} a_{mn}x_n - M_0 \\ \alpha_m \not< \phi, \quad & M_m = \sum_{n=1}^{n=m} a_{mn}x_n \end{aligned}\right\} \quad . \quad . \quad . \quad (14)$$

The equilibrium equation (7) becomes:—

$$\sum_{n=1}^{n=p} x_n \sin \alpha_n = 0 \quad . \quad . \quad . \quad (15)$$

Equation (9) is unchanged, and the remarks in (*a*) and (*b*) still apply.

(*d*) *Asymmetric Loadings*

With an asymmetric loading of any type, the method of procedure will be as in (*a*), (*b*) and (*c*), but twice the number of unknowns will be required, and integration must be carried out over the whole perimeter (instead of half as previously). For the loading shown in Fig. 17 the equilibrium equations will be:—

$$\left.\begin{aligned} \sum_{n=1}^{n=2p} x_n \sin \alpha_n &= \frac{P}{l} \\ \sum_{n=1}^{n=2r} x_n \cos \alpha_n &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (16)$$

and

$$\sum_{n=1}^{n=2p} x_n = \frac{Pb_1}{lr}$$

where b_1 is offset of P from centre of frame. Three Lagrange multipliers will be required and equation (9) will become:—

$$\lambda_1 \sin \alpha_n + \lambda_2 \cos \alpha_n + \lambda_3 = 2K_1 x_n + 2K_2 \sum_{m=n}^{m=2p} \frac{M_m}{I_m} a_{mn} \quad (17)$$

There will be $2p+2$ equations to solve, and the method is beginning to lose its simplicity. It will be better therefore to break the loading down into a combination of a symmetrical load and a moment load of the type given in (*e*) if this is possible without too great a loss of accuracy.

(*e*) *A moment load or a tangential load applied at the point diametrically opposite the centre of the cut-out* (Fig. 18)

The shear on the two sides of the frame is now anti-symmetric, so only p variables are required. M_m is unchanged, but there will be *two* equilibrium equations for each loading.

Moment Loading:—

$$\left.\begin{aligned} \sum_{n=1}^{n=p} x_n &= \frac{M_0}{2rl} \\ \sum_{n=1}^{n=p} x_n \cos \alpha_n &= 0 \end{aligned}\right\} \quad . \quad . \quad (18)$$

Tangential Loading:—

$$\left.\begin{aligned} \sum_{n=1}^{n=p} x_n &= 0 \\ \sum_{n=1}^{n=p} x_n \cos \alpha_n &= \frac{T}{2l} \end{aligned}\right\} \quad . \quad . \quad (18a)$$

Equation (9) will thus contain two Langrange operators, and for both moment and tangential loadings it will become:—

$$\lambda + \mu \cos \alpha_n = 2K_1 x_n + 2K_2 \sum_{m=n}^{m=p} \frac{M_m}{I_m} a_{mn} \quad . \quad (19)$$

The case when the frame is completely filled with a shear web (*see* Fig. 27), is of considerable practical value. The bending terms disappear from equation (19), which could then be rewritten

$$x_n = a + b \cos \alpha_n$$

where a and b are constants. These are easily determined from considerations of overall equilibrium as follows.

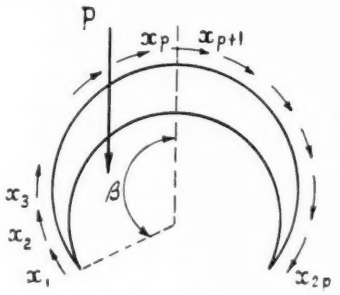

FIGURE 17. Asymmetric loading showing reacting shear ($2p$ unknowns. *See* Section 15(*d*)).

Moment Loading:—

$$\int_0^{\beta} (a + b \cos \alpha) r^2 d\alpha = \frac{M_0}{2}$$

and

$$\int_0^{\beta} (a \cos \alpha + b \cos^2 \alpha) r d\alpha = 0,$$

giving

$$a\beta + b \sin \beta = \frac{M_0}{2r^2}$$

$$a \sin \beta + \frac{b}{2}\left(\beta + \sin \frac{\beta}{2}\right) = 0,$$

which yields

$$a = -\frac{M_0}{4r^2}\left(\frac{\beta + \frac{1}{2}\sin 2\beta}{\left(\sin^2 \beta - \frac{\beta^2}{2} - \frac{\beta \sin 2\beta}{4}\right)}\right)$$

$$b = \frac{M_0 \sin \beta}{2r^2}\left(\frac{1}{\sin^2 \beta - \frac{\beta^2}{2} - \frac{\beta \sin 2\beta}{4}}\right).$$

Tangential Loading:—

$$\int_0^{\beta} (a + b \cos \alpha) r^2 d\alpha = Tr$$

$$\int_0^{\beta} (a \cos \alpha + b \cos^2 \alpha) d\alpha = \frac{T}{2r},$$

which similarly yields

$$a = \frac{T}{2r}\frac{\left(\sin \beta - \frac{\beta}{2} - \frac{1}{4}\sin 2\beta\right)}{\left(\sin^2 \beta - \frac{\beta^2}{2} - \frac{\beta \sin 2\beta}{4}\right)}$$

$$b = \frac{T}{2r}\frac{\sin \beta - \beta}{\left(\sin^2 \beta - \frac{\beta^2}{2} - \frac{\beta \sin 2\beta}{4}\right)}.$$

When $\beta=\pi$, *i.e.* a complete bulkhead, these relations yield:—

Moment Loading. Shear flow is $\frac{M_0}{2\pi r^2}$.

Tangential Loading. Shear flow is $\frac{T}{\pi r}(\frac{1}{2}+\cos\alpha)$.

These results agree with equation (3).

16. An Alternative Method of Solution for Moment and Tangential Loading

If the distribution of the reacting shear is known for a radial load P applied at a point, the distribution for a tangential load T or a moment load M_0 applied at the same point may be deduced directly. The method is due to Goodey (Ref. 8, Fig. 5), and is also referred to by Lewis (Ref. 11, pp. 6-8). The shear flow for an applied radial load is shown to be the differential coefficient of the shear flow for an applied tangential load. Conversely, the shear flow for a tangential load may be deduced from that for a radial load by a single integration with adjustment for the arbitrary constant.

In practice, the shear flow for the radial load is plotted against θ, integrated, multiplied by the ratio T/P and the axis adjusted for the constant of integration. This constant can be estimated by considerations of overall equilibrium. More simply, it can be determined by inspection if the value of the shear flow at any one point on the perimeter can be deduced from general considerations.

An exactly similar procedure can be applied to bending moments, axial loads and web shears. (This was also pointed out by Goodey[8].) The exercise of

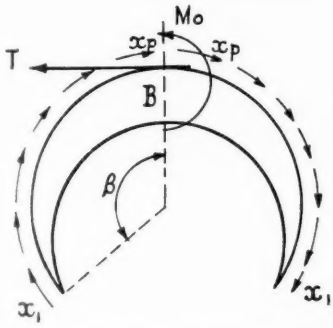

FIGURE 18. Moment or tangential loading at B showing anti-symmetric shear (p unknowns only. *See* Section 15(*e*)).

a little ingenuity generally suffices to determine the arbitrary constants without much calculation.

For example, quoting from Wise (Ref. 9, equation (16)), the axial load in the frame due to a radial load P is given by

$$N=\frac{P}{2\pi}\left[\frac{3}{2}\cos\theta+(\pi-\theta)\sin\theta\right].$$

Multiplying by T/P and integrating with respect to θ, gives for a tangential load T,

$$N=\frac{T}{2\pi}\left[\frac{1}{2}\sin\theta-(\pi-\theta)\cos\theta+A\right],$$

where A is an arbitrary constant. But at $\theta=0°$, the point of application of the load, it is obvious from Fig. 6 of Ref. 9 that by symmetry $N=-\frac{1}{2}T$ (or at $\theta=\pi$, $N=+\frac{1}{2}T$). This gives $A=0$, and hence

$$N=\frac{T}{2\pi}\left[\frac{1}{2}\sin\theta-(\pi-\theta)\cos\theta\right],$$

agreeing with equation (18) of the same reference.

For a moment load M_0 the shear flow for a tangential load is differentiated twice with respect to θ, and the result is added to the original shear flow for the tangential load. On multiplying by the ratio $M_0/(rT)$, the shear flow for a moment load is obtained. A similar procedure can be applied to bending moments, axial loads, and web shears in the frame.

Once again, quoting from Wise (Ref. 9, equation (18)) the bending moment M due to a tangential load T is given by

$$M=\frac{Tr}{2\pi}\left[(\pi-\theta)(1-\cos\theta)-\frac{3}{2}\sin\theta\right]$$

hence $$\frac{\partial^2 M}{\partial\theta^2}=\frac{Tr}{2\pi}\left[(\pi-\theta)\cos\theta-\frac{1}{2}\sin\theta\right]$$

and for the moment load M_0,

$$M=\frac{M_0}{2\pi}\left[(\pi-\theta)-2\sin\theta\right],$$

agreeing with equation (13) of the same reference.

The method appears unsuitable for use with shears obtained graphically by consideration of a few points only, for differential coefficients calculated as slopes of such curves are notoriously unreliable. (If however the second differential coefficient is small compared with the original term, this objection disappears.)

In practical calculations the first differential coefficient of the shear from the radial load would be used in place of the second differential coefficient of the shear from the tangential load.

17. Restraint from the Wing

The effect of restraint from the wing can readily be estimated by standard engineering methods, applied to the results of an analysis based on any of the methods described in this paper. One simple procedure is as follows.

If the restraint occurs at points defined by δ_1, δ_2, use the selected method to determine the bending moment in the frame for unit load applied at δ_1 in the direction of the restraint, and similarly for unit load applied at δ_2. Determine also the bending moment due to the given system of external loading.

Double integration then yields the deflection at δ_1 and also at δ_2 for, (*a*) an assumed loading of X_1 at δ_1, (*b*) an assumed loading of X_2 at δ_2, (*c*) the given external loading. Equating the total deflection at δ_1 and δ_2 separately to zero gives two equations for X_1 and X_2.

The effect of a restraint on the "leg" of a partial bulkhead is to relieve the bending stresses at the top of the leg (P and P' in Fig. 14). Normally only a portion of this calculated relief could be safely applied as the restraint is unlikely to be absolute.

PART III

A DISCUSSION OF THE UNDERLYING PARAMETERS WHICH DETERMINE THE STRESS DISTRIBUTION

18. Introduction

Analysis by the "strip method" of Part II throws considerable light on the nature of the underlying parameters which determine the stress distribution in the skin and the frame. In the succeeding sections, these parameters are discussed and the predictions of the "strip" theory are compared with the findings of other investigators.

In general, agreement is good, but some previously published results are shown to be in error, as important factors have been ignored or results have been plotted against incorrect parameters.

19. Variation with I, r, L, t and t'

It was shown in Section 14 that the degree of distortion of the skin shear distribution pattern depended on the ratio of the strain energy of the frame in bending to the strain energy of the skin in shear. From Section 14 these are respectively

$$\frac{l}{2E} \simeq \frac{M_n^2}{I_n} \text{ and } \frac{lL}{2tG} \simeq x_n^2.$$

Dimensionally

$$M_n \propto l \times \text{geometrical offset} \times x_n$$
$$\propto r^2 x_n.$$

Hence, dimensionally, strain energy of frame in bending

$$\propto \frac{r^5}{EI} x_n^2,$$

and strain energy of skin in shear

$$\propto \frac{Lr}{2tG} x_n^2.$$

The ratio of these yields a non-dimensional parameter $Gtr^4/(EIL)$. For small values of this parameter, the skin shear will be "basic." For high values the skin shear will be distorted.

This parameter is identical with the parameter A/B of Ref. 7. Moreover the curves in Ref. 7 show that for $A/B=0$ the skin shear is basic, and for increasing values the skin shear pattern becomes increasingly distorted. There is thus good agreement between Ref. 7 and the theory of Part II on this point.

There is another such parameter common to Ref. 7 and the present paper; this is representative of the importance of the skin and stringers in carrying end load. From Appendix IV, the strain energy of the skin and stringers over a length L of the fuselage is proportional to $\frac{L^3 x_n^2}{Elt'} \propto \frac{L^3 x_n^2}{Ert'}$.

Likewise the strain energy of the frame in bending is proportional to $x_n^2 r^5/(EI)$. The ratio of these gives the non-dimensional parameter $L^3 I/(r^6 t')$, which is the reciprocal of the parameter A in Ref. 7. Small values of A thus indicate high stringer energy, with consequent relief to shear distortion (*see* Section 14(*g*) and Appendix IV). Large values of A represent low stringer energy with little relief to shear distortion.

So far it has been implicitly assumed that when the frame bending energy is negligible (rigid frame) the stress distribution is controlled by the skin shear energy and that provided that the stringers are distributed evenly round the frame, their energy has no effect on the shear distribution. This is in line with engineering experience, and with the results of Ref. 7, but it is more satisfactory to demonstrate mathematically that this is indeed the case. Such a demonstration is given in Appendix VII. (It is shown to be only approximately true for an incomplete frame but strictly true for a complete frame.)

The following is thus obtained. At small values of A/B, there is little shear distortion, and the shear pattern is relatively insensitive to variations in A (provided that the end load carrying material is distributed evenly round the frame). At high values of A/B, shear distortion is considerable, and increases with increasing values of A. This is fully in agreement with the curves plotted in Ref. 7.

With the aid of Ref. 7, the following, therefore, can be deduced:—

(*i*) When A/B is small (less than 1·0, say), the frame can be treated as rigid.

(*ii*) When A/B is larger (not greater than 10, say), the frame is slightly flexible. Stringer energy (or more strictly, energy of skin and stringers in end load) can be neglected in strain energy calculations. Analysis based on engineering theory may be used if desired, the maximum skin shear being then underestimated (by about 10 per cent. when A/B is 10) and the maximum bending moment being overestimated (by about 15 per cent. when A/B is 10).

(*iii*) When A/B is large (of the order of 200 upwards, say), the frame is truly flexible. Stringer energy is important (unless A is large, of the order of 10^6, say). Analysis based on engineering theory will lead to substantial errors.

A quantitative character has thus been given to the definition of Section 14(*j*). It should be noted that these criteria apply primarily to complete frames and that frames with large cut-outs will tend to develop "flexibility" at smaller values of A/B.

The non-dimensional parameters A and A/B appear to be of so fundamental a nature that they should figure (directly or indirectly) in all sound frame analyses. It is interesting to look for them in other published work.

Starting with Ref. 8, p. 848; in the derivation of equation (D7) Goodey speaks of two non-dimensional parameters "c" and "e" which represent the

"fundamental characteristics of the structure." Using Goodey's own notation, these are respectively

$$c = 3m^3\left(1+\frac{1}{r}\right)\phi \quad \text{where } m=\frac{R}{L};\quad \phi=\frac{R^3K_xt}{I_f}.$$

$$K_x=\frac{t'}{t}\ (t' \text{ being in the notation of the present paper})$$

$$e = 3m^2(2K_x(1+\nu)-\nu).$$

These yield

$$c=\frac{3R^6t'}{L^3I_f}\left(1+\frac{1}{r}\right)\simeq\frac{6R^6t'}{L^3I_f}=6A,$$

$$\text{and } e=\frac{3R^2}{L^2}\left(\frac{t'}{t}\frac{E}{G}-\nu\right) \text{ since } \frac{E}{G}=2(1+\nu),$$

$$i.e.\ e\simeq\frac{3R^2}{L^2}\frac{t'}{t}\frac{E}{G}=3B,$$

where ν is Poisson's ratio and r a constant representing a comparison between frame stiffnesses, r being unity when all the frames have equal stiffnesses. Examination of equation (D12) in Ref. 8 shows that the shear flow is dependent on e/c and also on c, but as there are no curves showing the variation of the shear flow with these constants, it is not possible to deduce their effect so readily as in Ref. 7.

Equations (D7) and (D12) of Ref. 8 refer to a short length of fuselage. For a long fuselage, Goodey introduces a parameter λ which is dependent on his c and e. His final expression for the shear flow is dependent on λ and hence also on c and e, but once again the exact method of dependence is not easy to determine without complicated algebraic manipulation or parametric plotting.

The parameters A and B also appear in Ref. 4. Hoff points out their importance, but with rather different emphasis (*see* Introduction to Ref. 4). Also, Hoff's A and B have values six times greater numerically than the A and B of Ref. 7 and the present paper.

Reference 1 also uses a non-dimensional parameter d which is defined as $kR^3/(EI)$ (notation of Ref. 1). k is later defined as RtG/L, giving $d=GtR^3/(EIL)$, entirely equivalent to the A/B of the present paper or of Ref. 7. Fig. 1 of Ref. 1 also gives numerical values of d, above which a frame is "flexible." These values agree well with those given in this paper.

The parameter A does not, however, appear. This indicates that the results of Ref. 1 apply only to fuselages where the energy of the skin and stringers in end load is negligible. The effect of neglecting A is shown in Fig. 29, which is a cross plot from the curves of Ref. 7. The quantity λ_1 is defined as

$$\lambda_1=\frac{\text{Maximum bending moment (or skin shear, etc.) for a given value of } A}{\text{Maximum bending moment (or skin shear, etc.) for } A=10^6}$$

$A=10^6$ may be taken as "strain energy of skin and stringers negligible," *i.e.* skin and stringers stiff in end load. The curves show clearly, that unless this energy is negligible, large errors can occur by neglecting it.

Beskin[5] used a parameter $AL/[(1-\eta)tR^2]$ (in his notation A is the total cross-sectional area of the frame). Since $I=A\eta^2R^2$, the parameter may be written $IL/[R^4t\eta^2(1-\eta)]$ or approximately $(1/\eta^2+1/\eta)IL/(R^4t)$. Ignoring the eccentricity, this gives a numerical multiple of the reciprocal of the parameter A/B.

Beskin uses another parameter $R^2h/(L^2t)$ (again in his own notation, h being equivalent to t' in this paper). This is a numerical multiple of the B of Ref. 7. Although Beskin states that this second parameter has little effect on the stress distribution, since he is primarily investigating the effects of eccentricity, his statement must be construed as meaning that, in the range he investigated, the effects of eccentricity on the stress distribution were not sensitive to variations in B. (Further discussion of the effect of eccentricity will be found in Section 20.)

It is gratifying, therefore, that so simple an analysis as the present paper offers should agree with other more elaborate investigations, including one so basic as Goodey's, as to the nature of the fundamental constants that determine the skin shear.

Lewis[11] produces curves for skin shear based on Goodey's analysis in Ref. 8. These include some interesting curves showing the variation of skin shear with I, L, t and t' (in the notation of the present paper) and also showing the effect of keeping the I of the loaded frame constant but varying the I of adjacent frames. He concludes that within reasonable limits the skin shear is not sensitive to quite large variations in these quantities. This is in general agreement with the present paper but seems to require some qualification.

Thus, consider variation with I. The sensitivity is small if A/B is less than 1·0 (say), as in this range the bending energy of the frame is trivial. It can also be small when A/B is large (greater than 1,000, say) as in this range stringer energy can become important compared to bending energy. Maximum sensitivity would therefore be expected in intermediate ranges. For example, in the neighbourhood of $A/B=200$, local variations in I of the type used in Appendix III could conceivably be highly significant. Also, when A/B is large (greater than 1,000, say) they could again be significant if stringer energy and bending energy happened to be of about the same magnitude. But the point requires examination using actual numerical examples.

Similar remarks apply to variations in L, t and t'. The range of greatest sensitivity to L (average frame spacing) would probably be A/B greater than 1,000 with A not greater than 10^4, *i.e.* flexible frames with high stringer energy.

Section 6 of the "Conclusions" and Fig. 21 (of Ref. 11) also seem to require modification. Fig. 21 presents a plot of I against r (in the notation of the present paper) and states that engineering beam theory may be used if I falls above the line. It has already been shown, however, that the fundamental parameter that determines this is $Gtr^3/(EIL)$. To plot I against r is therefor misleading. I should be plotted against $Gtr^3/(EIL)$ or against c/e (Goodey's constants). Selecting numerical examples at random gives the values in Table I.

From Fig. 21 of Ref. 11, all the frames would have been treated as rigid, but Ref. 7 and Table I indicate that this is incorrect except for the first.

In the discussion of the present section, L has been used in two senses—to represent frame spacing, when referring to the work of other investigators, and to represent "the length along the fuselage in which the shear pattern reverts to basic" when referring to the results previously deduced in the present paper. There is an apparent discrepancy, but this is not serious as the argument of the present section is based on dimensions, and since the distorted shear reverts to basic in a small integral number of bays, the two senses are dimensionally similar. For calculating numerical values of A and A/B, however, L should be taken as meaning "frame spacing."

One other point arises in connection with the effect of stringers on shear distribution. Hoff has examined the effect of the applied load being reacted partly by

TABLE I

r	L	t	t'	I	$\frac{A}{B}=\frac{Gtr^4}{EIL}$	$A=\frac{r^2t'}{IL^3}$	*Shear cone factor from Ref. 7*	*Bending moment reduction factor from Ref. 7*
20	20	0·036	0·07	8	14	70	1·2 (Fig. 6)	0·80 (Fig. 3)
50	50	0·048	0·10	10	240	1,250	2·0 (Fig. 21)	0·55 (Fig. 18)
70	70	0·064	0·12	12	720	3,400	2·8 (Fig. 21)	0·45 (Fig. 18)

(Values of A and A/B are approximate only. Figure references are to Ref. 7.)

radial and tangential bending of the stringers. Figs. 13 to 15 of Ref. 4 show the magnitude of this effect. It is small if A is less than 10^6. (Note that A has values six times greater numerically than in the present paper.)

Goodey[8] also considered the effect of bending and torsion of the stringers, although he ultimately ignored it in the interests of simplicity.

The present analysis could easily be extended to include this effect. It is simpler, however, to ignore it in actual calculation and to correct the peak values afterwards, using Hoff's curves. It will rarely be significant.

20. Variation with Eccentricity (*)

Most of the published analyses on fuselage frames treat the skin line as coincident with the centre line of the frame. Beskin[5], however, has investigated the effect of eccentricity, *i.e.* the offset of the skin line from the frame centre line.

Beskin calls the skin radius R and the radius of the frame centre line $(1-\eta)R$, where η is the eccentricity. He takes A as the area of cross section of the frame, and $A\eta^2R^2$ as the moment of inertia of the cross section of the frame. As already explained in Section 19, he finds that the effect of eccentricity depends on two parameters, sensitivity to one of these being small.

A cross plot* of Beskin's curves is given in Fig. 30. This shows the effect of eccentricity on maximum frame stresses, plotted against the parameter A/B (notation of the present paper), values of A/B having been obtained from Beskin's parameter $AL/[(1-\eta)tR^2]$ (Beskin's notation) by using the conversion:—

$$A/B = \text{"Reciprocal of Beskin's parameter"} \times 0{\cdot}4\left(\frac{1}{\eta^2}+\frac{1}{\eta}\right).$$

λ_η is defined as

$$\lambda_\eta = \frac{\text{Maximum bending moment (or web shear, etc.) for a given value of } \eta}{\text{Maximum bending moment (or web shear, etc.) for } \eta=0}$$

Surprisingly, the cross plot shows that for a rigid frame loaded radially the maximum bending moment is independent of the eccentricity. This case can readily be analysed by routine strain energy methods, and such an analysis is given (for convenience) in the second part of Appendix I of the present paper. It will be seen that the maximum bending moment comes out as $3Pr(1-\eta)/(4\pi)$ or approximately $0{\cdot}24(1-\eta)Pr$, whereas Beskin's curves give $0{\cdot}24\,PR$, both r and R being skin radius. The author is not able, at present, to offer an explanation of the discrepancy.

Beskin does not show the effect of eccentricity on skin shear distribution. Hoff[4], however, presents curves showing the effect of eccentricity on moment reduction factor and skin shear concentration factors, plotted against his parameter A. Thus, from Figs. 5 and 6 of Ref. 4, when $A=10^4$, varying η from zero to 0·075 makes little difference to the shear concentration factor, but increases the moment reduction factor by about 10 per cent. For $A=10^6$, varying η from zero to 0·075 decreases the shear concentration factor to about half, and increases the moment reduction factor by about 25 per cent.

To interpret these results, it is necessary to decide whether the skin radius is kept constant and the frame radius diminished, or *vice versa*. Keeping the skin radius constant, the results mean that increasing eccentricity decreases the skin shear distortion and increases the frame bending moment.

Predictions of the effect of eccentricity from the theory developed in Part II are rather unsatisfactory. Thus, keeping the skin radius constant and allowing the frame radius to decrease, but also allowing the frame moment of inertia to increase as in Beskin's article, has the following effects: (*a*), terms in equation (9) from skin shear will remain unaltered, (*b*), terms in equation (9) from bending energy will be decreased on two counts—the bending moment is reduced as already explained and the moment of inertia is increased. The shear terms therefore become relatively more important with increasing eccentricity and hence there is less shear distortion, with a consequent increase in the bending moment. Whether the final effect is a net increase or a net decrease seems difficult to predict. But the decrease in shear distortion at any rate is in agreement with Hoff's curves. Moreover, the effect should be more marked at high values of A than at low values, for low values of A indicate high stringer energy, in which case

*Minor irregularities have not been "faired in." They would probably disappear if higher harmonics were considered in the original solution.

small changes in frame bending energy are less significant. This, too, is in keeping with Hoff's curves.

21. Variation with Number of Fields

Hoff[4] has investigated the effect of varying the number of fields between the loaded frame and the support. He finds maximum distortion of the skin shear pattern with one field, and decreasing distortion with increasing number of fields. The maximum bending moment increases with the number of fields. The curves showing these effects tend to flatten off after three or four fields.

Other investigators have pointed out that increasing the number of fields above three has little effect on the maximum shear and bending moment (*see* Ref. 7, Fig. 6 and Ref. 11, p. 5).

Goodey's analysis[8] includes the effect of fields on the side of the loaded frame remote from the support (*i.e.* to the right of P in Fig. 1 of the present paper). Compatibility demands that there shall be self-equilibrating stress systems in this field, but their magnitude appears to be small in general.

The present theory explains the significance of the number of fields as follows. Variation of the number of fields gives variation in L, the length in which the loaded shear reverts to basic. When A/B is small, skin shear energy is paramount, and the distribution is not sensitive to L (*see*, however, Appendix VII). When A/B is in an intermediate range, where shear and bending energies are comparable, the shear distribution will be sensitive to L and therefore to the number of fields. When A/B is large, stringer energy is becoming important compared with bending energy and once again the final shear distribution will depend on L and therefore on the number of fields.

This does not explain the generally observed phenomenon that sensitivity is small once the number of fields exceeds three or four. It is suspected that this is more of a coincidence, based on the range inside which practical aircraft designers work, than a fundamental theoretical necessity. It is brought about by the fact that the bending energy produced in succeeding frames by change of the shear flow at these frames is less than the additional shear energy in the skin if the distorted shear had continued unchanged.

The preceding considerations suggest one point of practical importance. If a short length of fuselage is used on test to represent a longer fuselage, provided that there are at least six fields and the loaded frame is two fields from the open end, the maximum measured shear flows and bending stresses should adequately represent the maximum values actually occurring on the longer fuselage.

22. Extensional and Shearing Rigidities of the Frames

The present analysis has so far taken no account of the extensional and shearing rigidities of the frames. These will be important when the strain energy of the frame in axial load or of the web in shear is large compared with the bending energy of the frame.

Dimensionally, strain energy from axial load depends on $(lx_n)^2[l/(ES)]$, *i.e.* $r^3x_n^2/(ES)$, and due to web shear $(lx_n)^2[l/(GS')]$, *i.e.* $r^3x_n^2/(GS')$. From Section 19 strain energy in bending depends on $r^3x_n^2/(EI)$. The relevant ratios are thus $I/(Sr^2)$ for axial load and $I/[(G/E)S'r^2]$ for web shear. Hoff[4] calls the first ratio C, and the second is equivalent to his CD, where he defines D as $S/(GS'/E)$.

The present theory thus predicts that the axial and shearing rigidities will be important as C and CD increase. In Ref. 4, Figs. 9 and 10, Hoff shows the effect of variation of C (keeping D constant) and D (keeping C constant). The shear concentration surprisingly goes down with increasing C, which appears to be contrary to the present theory. The apparent discrepancy arises perhaps from the manner of variation of the parameters, which Hoff does not specify, except that he keeps A and B constant. If I and r are kept constant while S varies, increase of C increases the total strain energy of the frame and hence the shear concentration factor. If, however, I, r and S all vary it might be possible for an increase of C to be accompanied by a decrease in the total strain energy of the frame compared with the strain energy of the skin in shear, and this would give a decrease in the shear concentration.

In general, the effect of including shearing and extensional rigidities of the frame is small, but where there is reason to believe these effects are significant, terms giving the strain energy of the frame in axial load and the web in shear in terms of x_1 to x_p can easily be included in equations (6) to (9) of Section 14.

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APPENDIX I

STRAIN ENERGY ANALYSIS OF A COMPLETE CIRCULAR FRAME OF CONSTANT CROSS SECTION, LOADED RADIALLY AT ONE POINT, THE REACTING SHEAR BEING DISTRIBUTED SINUSOIDALLY

The radial load acts at A (Fig. 8), the frame is assumed cut at B and redundancies N, V, M_2 are introduced. By symmetry $V=0$. At the point defined by θ, the shear flow is $[P/(\pi r)]\sin\theta$. Hence at the point defined by α, the bending moment in the frame is

$$M = M_2 - Nr(1+\cos\alpha) + \int_{\alpha}^{\pi} \left(\frac{P}{\pi r}\sin\theta\right) r^2 [1-\cos(\theta-\alpha)]\, d\theta$$

$$= M_2 - Nr(1+\cos\alpha) + \frac{Pr}{\pi}\{1+\cos\alpha - \tfrac{1}{2}(\pi-\alpha)\sin\alpha\}$$

where positive M gives tension in the inner flange.

Then $\frac{\partial M}{\partial M_2}=1$ and $\frac{\partial M}{\partial N}=-r(1+\cos\alpha)$.

From accepted strain energy theory,

$$\int_0^\pi M\frac{\partial M}{\partial M_2}d\alpha=0 \qquad \text{and} \int_0^\pi M\frac{\partial M}{\partial N}d\alpha=0.$$

Combining these four conditions gives

$$\int_0^\pi Md\alpha=0 \qquad \text{and} \int_0^\pi M\cos\alpha\,d\alpha=0$$

and hence

$$\left.\begin{aligned} M_2\pi-Nr\pi+\frac{Pr}{2}&=0\\ -\frac{Nr\pi}{2}+\frac{3Pr}{8}&=0\end{aligned}\right\}$$

i.e. $N=\frac{3P}{4\pi}$ $\qquad M_2=\frac{Pr}{4\pi}$

(N positive, tension. M_2 positive, tension on inner flange)

Finally $M=\frac{Pr}{2\pi}\{1+\frac{1}{2}\cos\alpha-(\pi-\alpha)\sin\alpha\}$

which agrees with equation (16) of Ref. 9.

NOTE (*i*) Where the shear flow is not capable of analytic expression, numerical values of M and $M\cos\alpha$ must be tabulated round the frame and the integrals

$$\int_0^\pi Md\alpha \quad \text{and} \quad \int_0^\pi M\cos\alpha\,d\alpha$$

must be evaluated graphically. The work is considerable, as to get numerical values of M and $M\cos\alpha$ at any one station requires graphical integration also. Terms involving M_2 and N can, however, be evaluated analytically throughout the calculation.

(*ii*) In the preceding, the skin line has implicitly been assumed coincident with the centre line of the frame. If this is not the case (*i.e.* if there is "eccentricity" (*see* Section 20), taking the skin radius as r and the frame centre line radius as r', and applying the redundancies N, M at the frame centre line, gives

$$\begin{aligned} M&=M_2-N(r'+r'\cos\alpha)+\\ &\quad+\int_\alpha^\pi\left(\frac{P}{\pi r}\sin\theta\right)[r-r'\cos(\theta-\alpha)]\,rd\theta\\ &=M_2-Nr'(1+\cos\alpha)+\\ &\quad+\frac{P}{\pi}[r(1-\cos\alpha)-\tfrac{1}{2}(\pi-\alpha)r'\sin\alpha].\end{aligned}$$

Proceeding as before:—

$$\frac{\partial M}{\partial M_2}=1, \qquad \frac{\partial M}{\partial N}=-r'(1+\cos\alpha),$$

$$\int_0^\pi M\frac{\partial M}{\partial M_2}d\alpha=0 \quad \text{and} \quad \int_0^\pi M\frac{\partial M}{\partial N}d\alpha=0,$$

giving

$$\int_0^\pi Md\alpha=0 \quad \text{and} \quad \int_0^\pi M\cos\alpha\,d\alpha=0,$$

which yields

$$\left.\begin{aligned} M_2\pi-Nr'\pi+P\left(r-\frac{r'}{2}\right)&=0\\ -\frac{Nr'\pi}{2}+P\left(\frac{r}{2}-\frac{r'}{8}\right)&=0\end{aligned}\right\}$$

Then, putting $1-\eta=\frac{r'}{r}$,

$$N=\frac{P}{4\pi}(3+4\eta), \qquad M_2=\frac{Pr}{4\pi}(1+2\eta),$$

and $M=\frac{Pr}{2\pi}(1-\eta)(1+\frac{1}{2}\cos\alpha+(\pi-\alpha)\sin\alpha).$

The maximum value of M (at $\alpha=0$) is

$$M=\frac{3Pr}{4\pi}(1-\eta).$$

This result is discussed in Section 20.

APPENDIX II

SOLUTION OF A COMPLETE CIRCULAR FRAME OF CONSTANT CROSS SECTION WITH A PROPPED FLOOR BEAM, LOADS BEING APPLIED THROUGH THE FLOOR BEAM

(*See* Figs. 9, 10 and 11 and Section 11)

The solution consists essentially of the determination of three redundancies, M_0, P and X by strain energy analysis, using curves such as those in the Royal Aeronautical Society Stressed Skin Data Sheets 06.03 series for a rigid frame, or N.A.C.A. T.N.1310 (*see* Ref. 7) for a flexible frame.

The bending moment at any point in the frame is given by

$$M=m_1(W-P\cos\alpha)+m_2X+m_3M_0+m_4P,$$

where m_1, m_2, m_3, m_4 are obtained from the curves.

Then

$$U_{\text{frame}}=\int_0^{2\pi r}\frac{(m_1W+m_2X+m_3M_0+m_5P)^2}{2EI_{\text{frame}}}\,ds$$

$$U_{\text{beam}}=\int_0^{b}\frac{(M_1-m_6P-M_0)^2}{2EI_{\text{beam}}}\,ds,$$

where U_{frame} and U_{beam} denote respectively the strain energy of the frame and the beam, s is the distance measured round the frame from some fixed point, M_1 is the free bending moment in the floor beam from the distributed load (*i.e.* M_1 is $(W/b)(bx-x^2)$), m_6 is a constant evaluated in Note (*a*) of this Appendix, and $m_5=m_4-m_4\cos\alpha$.

The foregoing ignores the effect of the horizontal components $P\cos\alpha$ and X on the bending moment. In general this is small; it can be included if desired.

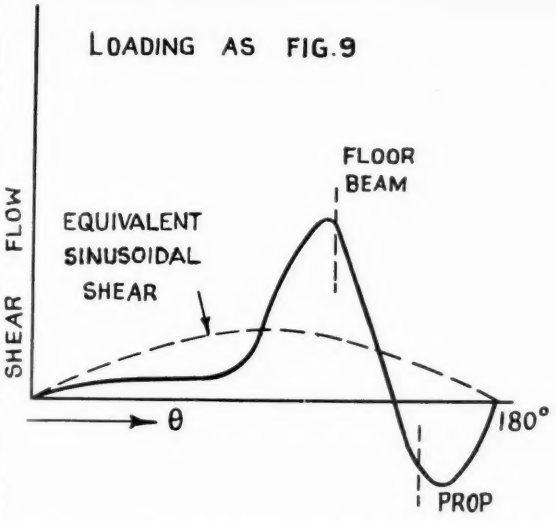

FIGURE 19. Typical skin shear distribution for a frame reinforced with a propped floor beam (*See* Fig. 9 and Appendix II).

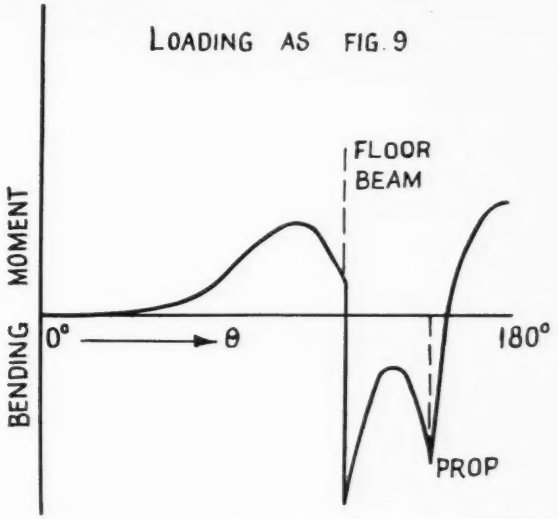

FIGURE 20. Typical bending moment distribution for a frame reinforced with a propped floor beam (*See* Fig. 9 and Appendix II).

Then if U is total strain energy, $U = U_{\text{frame}} + U_{\text{beam}}$, and

$$EI_{\text{frame}}\frac{\partial U}{\partial X} = W\int_0^{2\pi r} m_1 m_2 ds + X\int_0^{2\pi r} m_2^2 ds + M_0\int_0^{2\pi r} m_2 m_3 ds + P\int_0^{2\pi r} m_2 m_5 ds = 0,$$

$$EI_{\text{frame}}\frac{\partial U}{\partial M_0} = W\int_0^{2\pi r} m_1 m_3 ds + X\int_0^{2\pi r} m_2 m_3 ds + M_0\int_0^{2\pi r} m_3^2 ds + P\int_0^{2\pi r} m_3 m_5 ds - \frac{I_{\text{frame}}}{I_{\text{beam}}}\left[\int_0^b M_1 dx - P\int_0^b m_6 dx - M_0 b\right] = 0,$$

$$EI_{\text{frame}}\frac{\partial U}{\partial P} = W\int_0^{2\pi r} m_1 m_5 ds + X\int_0^{2\pi r} m_2 m_5 ds + M_0\int_0^{2\pi r} m_3 m_5 ds + P\int_0^{2\pi r} m_5^2 ds$$

$$\frac{I_{\text{frame}}}{I_{\text{beam}}}\left[\int_0^b M_1 m_6 dx - P\int_0^b m_6^2 dx - M_0\int_0^b m_6 dx\right] = 0.$$

These integrals are evaluated graphically and the equations solved for X, M_0 and P. By a further application of the curves already referred to, the frame can now be completely solved. Typical skin shear and frame bending moment distributions are shown in Figs. 19 and 20.

NOTES. (*a*) *Integrals containing m*
From Fig. 12,

$m_6 = x\cos\alpha$ from $x=0$ to $x=a$,
$m_6 = a\cos\alpha$ from $x=a$ to $x=b/2$,

whence

$$\int_0^b m_6 dx = 2\int_0^a x\cos\alpha dx + 2\int_0^{\frac{b}{2}} a\cos\alpha\, dx = a(b-a)\cos\alpha,$$

$$\int_0^b m_6^2 dx = 2\int_0^a x^2\cos^2\alpha\, dx - 2\int_0^{\frac{b}{2}} a^2\cos^2\alpha\, dx = \frac{a^2}{3}(3b-4a)\cos^2\alpha,$$

$$\int_0^b M_1 dx = \frac{W}{b}\int_0^b (bx - x^2)dx = \frac{Wb^2}{6},$$

$$\int_0^b M_1 m_6\, dx = \frac{2W\cos\alpha}{b}\int_0^a (bx^2 - x^3)dx + \frac{2W\cos\alpha}{b}\int_a^{\frac{b}{2}} a(bx - x^2)dx = \frac{Wa\cos\alpha}{6b}[a^3 - 2a^2 b + b^3].$$

(*b*) m_2 will consist of four parts arising respectively from the radial and tangential components of X on the left side and right side of the frame. In general two of these will be negligible. Similarly there will be four components in each of m_1, m_3, and m_4.

(*c*) Values of m_1, m_2, m_3 and m_4 should be tabulated from 0° to 180° and also values of their squares

and cross products. Integration can then be performed graphically. It is advisable to tabulate a considerable number of points, as there are numerous peaks on the cross product curves, and if some of these are overlooked considerable errors can result.

(*d*) A solution for a concentrated load at the centre of the floor beam is readily obtained from the preceding, the only change being in the value of M_1. Integrals containing M_1 must be recalculated, and the equations solved again, but the great bulk of the work stands unchanged.

(*e*) A direct solution for an asymmetric floor load would be more laborious, as there would be seven redundancies. It can, however, be broken up into a symmetric plus an anti-symmetric load, each of which involves only three redundancies. This reduces the computation work.

APPENDIX III

NUMERICAL SOLUTION FOR A PARTIAL BULKHEAD

(*See* Fig. 14 and Section 14(*d*))

$r=70$ in., $\alpha=55°$, $\beta=135°$, $L=55$ in., $K_1=Ll/(2tG)=0{\cdot}279\times10^{-2}$, $K_2=l/(2E)=9{\cdot}75\times10^{-7}$. Shear over $PP'=x_6\sin\theta$.

Length of strips $=16°$, *i.e.* $l=19{\cdot}5$ in., $t=0{\cdot}048$ in., $\alpha_1=127°$, $\alpha_2=111°$, $\alpha_3=95°$, $\alpha_4=79°$, $\alpha_5=63°$.

$a_{51}/l=42{\cdot}2$	$a_{41}/l=27{\cdot}0$	$a_{31}/l=15{\cdot}1$	$a_{21}/l=6{\cdot}4$	$a_{11}/l=2{\cdot}6$
$a_{52}/l=27{\cdot}4$	$a_{42}/l=15{\cdot}7$	$a_{32}/l=7{\cdot}7$	$a_{22}/l=4{\cdot}0$	
$a_{53}/l=16{\cdot}1$	$a_{43}/l=8{\cdot}4$	$a_{33}/l=5{\cdot}2$		
$a_{54}/l=9{\cdot}0$	$a_{44}/l=5{\cdot}8$			
$a_{55}/l=6{\cdot}5$				

$I_1=2{\cdot}5$, $I_2=8{\cdot}0$, $I_3=15{\cdot}0$, $I_4=27$, $I_5=46$ in.^{4}

$M_1=19{\cdot}5\ (2{\cdot}6\,x_1)$

$M_2=19{\cdot}5\ (6{\cdot}4\,x_1+4{\cdot}0\,x_2)$

$M_3=19{\cdot}5\,(15{\cdot}1\,x_1+7{\cdot}7\,x_2+5{\cdot}2\,x_3)$

$M_4=19{\cdot}5\,(27{\cdot}0\,x_1+15{\cdot}7\,x_2+8{\cdot}4\,x_3+5{\cdot}8\,x_4)$

$M_5=19{\cdot}5\,(42{\cdot}2\,x_1+27{\cdot}4\,x_2+16{\cdot}1\,x_3+9{\cdot}0\,x_4+6{\cdot}5\,x_5)$.

The strain energy of the skin over PP' is

$$\frac{70\times55}{2\times0{\cdot}048\times4\times10^6}\int_0^{55°}(x_6\sin\alpha)^2d\alpha$$

$$=1{\cdot}01\times10^{-2}x_6{}^2\left[\frac{\alpha}{2}-\frac{\sin2\alpha}{4}\right]_0^{55°}=2{\cdot}47\times10^{-3}x_6{}^2.$$

The equilibrium equation is

$$70\int_0^{55°}x_6\sin^2\alpha d\alpha+19{\cdot}5\sum_{n=1}^{n=5}x_n\sin\alpha_n=500,$$

i.e.
$$0{\cdot}88\,x_6+\sum_{n=1}^{n=5}x_n\sin\alpha_n=25{\cdot}7.$$

Minimising by Lagrange's method (*see* Appendix VI).

For x_6 $\quad 0{\cdot}88\,\lambda=4{\cdot}94\times10^{-3}x_6$,

i.e. $x_6=178\,\lambda$

For x_1 to x_5,

$$\lambda\sin\alpha_n=5{\cdot}58\times10^{-3}x_n+19{\cdot}5\times10^{-7}\sum_{m=n}^{m=5}\frac{M_m}{I_m}a_{mn}\quad .\qquad (a)$$

$$\text{Let } Q_n=\sum_{m=n}^{m=5}\frac{M_m}{I_m}a_{mn}\times\left(\frac{1}{19{\cdot}5}\right)^2.$$

Dividing equation (*a*) by $19{\cdot}5^2\times10^{-7}$,

$$Q_n+7{\cdot}53\,x_n=1350\,\lambda\sin\alpha_n\quad .\quad .\qquad (b)$$

Values of Q_n are now calculated in tabular form (Table II).

This gives the following equations, by substitution into equation (*b*):—

$$\begin{aligned}
96{\cdot}2x_1&-51{\cdot}8x_2&+28{\cdot}4x_3&+14{\cdot}1x_4&+5{\cdot}95x_5&=1080\lambda\\
51{\cdot}8x_1&-39{\cdot}2x_2&+17{\cdot}1x_3&+8{\cdot}74x_4&+3{\cdot}86x_5&=1260\lambda\\
28{\cdot}4x_1&-17{\cdot}1x_2&+17{\cdot}53x_3&+4{\cdot}95x_4&+2{\cdot}27x_5&=1345\lambda\\
14{\cdot}1x_1&-8{\cdot}74x_2&+4{\cdot}95x_3&+10{\cdot}54x_4&+1{\cdot}27x_5&=1326\lambda\\
5{\cdot}95x_1&-3{\cdot}86x_2&+2{\cdot}27x_3&+1{\cdot}27x_4&+8{\cdot}45x_5&=1205\lambda\\
&&&&x_6&=165\lambda
\end{aligned}$$

$$0{\cdot}799x_1-0{\cdot}934x_2+0{\cdot}996x_3+0{\cdot}982x_4+0{\cdot}891x_5+0{\cdot}88x_6=25{\cdot}7.$$

Solution of the first five equations yields

$$x_1=-54{\cdot}7\,\lambda,\quad x_2=26{\cdot}1\,\lambda,\quad x_3=89{\cdot}6\,\lambda,\quad x_4=120{\cdot}0\,\lambda,$$
$$x_5=127{\cdot}1\,\lambda.$$

Substitution of these values in the last equation yields $\lambda=0{\cdot}0727$, and hence $x_1=-3{\cdot}07$, $x_2=1{\cdot}46$, $x_3=5{\cdot}02$, $x_4=6{\cdot}73$, $x_5=7{\cdot}15$, $x_6=10{\cdot}0$. (All these have been calculated to slide rule accuracy only.)

From these values the bending moment is obtained by substituting in the expressions for P_1 to P_5. Axial load and web shear at α_m are given respectively by

$$19{\cdot}5\sum_{n=1}^{n=m}x_n\cos(\alpha_n-\alpha_m)\text{ and }19{\cdot}5\sum_{n=1}^{n=m}x_n\sin(\alpha_n-\alpha_m),$$

with correspondingly adjusted formulae over the region PP'.

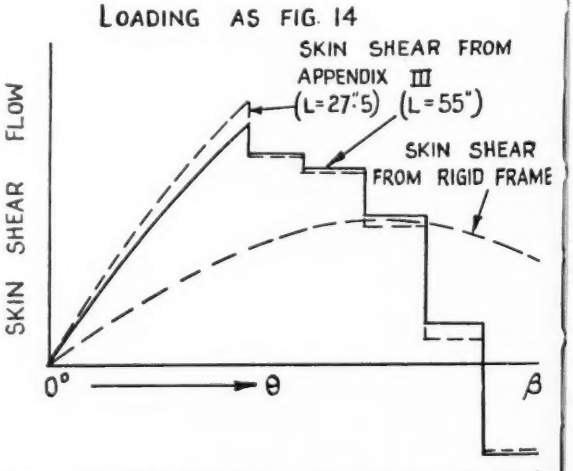

FIGURE 21. Skin shear flow as given by Appendix III for partial bulkhead, compared with skin shear flow from engineering theory.

A small refinement must be used in the calculation of the axial load. In the summation, the term giving the increment of axial load at the centre of the m^{th} strip due to shear on the m^{th} strip must clearly be halved. (This is also true of bending moment and web shear, but the increments are respectively trivial and zero.)

It will be noticed that the shear at the cut end of the frame is not zero. This presupposes the existence of an edge member capable of carrying end load. In practice such a member will almost certainly be present, but if its cross-sectional area a_1 is small, its strain energy (given by $L^3x_1^2/(6Ea_1)$) should be included in the calculation. This will have the effect of reducing x_1.

Figure 21 shows the skin shear plotted against θ. Skin shear from engineering theory is included for comparison. The effect of varying L is also shown. When L is halved, the maximum shear is increased by about 10 per cent. When L is doubled or quadrupled, the maximum shear is decreased by about 9 per cent. and 17 per cent., respectively. (For clarity, these latter values were not plotted.) The considerations of Section 14(*i*), however, enable quite narrow limits to be put on the allowable variation of L, and the numerical results given thus support the statement in Section 14(*i*) that the maximum shear will not be very sensitive, in general, to variations of L within these limits.

APPENDIX IV

EFFECT OF THE INCLUSION OF THE STRAIN ENERGY OF THE SKIN AND STRINGERS IN END LOAD

The calculation of Appendix III is modified as follows. Let $t'=0{\cdot}06$ in. Assume that the total end load carrying area per strip is concentrated at the

TABLE II

	x_1	x_2	x_3	x_4	x_5
	$n=1$:				
	$2{\cdot}6\times\frac{2{\cdot}6}{2{\cdot}5}=2{\cdot}7$	—	—	—	—
	$6{\cdot}4\times\frac{6{\cdot}4}{8}=5{\cdot}1$	$4{\cdot}0\times\frac{6{\cdot}4}{8}=3{\cdot}2$	—	—	—
	$15{\cdot}1\times\frac{15{\cdot}1}{15}=15{\cdot}2$	$7{\cdot}7\times\frac{15{\cdot}1}{15}=7{\cdot}7$	$5{\cdot}2\times\frac{15{\cdot}1}{15}=5{\cdot}2$	—	—
	$27{\cdot}0\times\frac{27{\cdot}0}{27}=27{\cdot}0$	$15{\cdot}7\times\frac{27{\cdot}0}{27}=15{\cdot}7$	$8{\cdot}4\times\frac{27{\cdot}0}{27}=8{\cdot}4$	$5{\cdot}8\times\frac{27{\cdot}0}{27}=5{\cdot}8$	—
	$42{\cdot}2\times\frac{42{\cdot}2}{46}=38{\cdot}7$	$27{\cdot}4\times\frac{42{\cdot}2}{46}=25{\cdot}2$	$16{\cdot}1\times\frac{42{\cdot}2}{46}=14{\cdot}8$	$9{\cdot}0\times\frac{42{\cdot}2}{46}=8{\cdot}25$	$6{\cdot}5\times\frac{42{\cdot}2}{46}=5{\cdot}95$
Q_1	$=88{\cdot}7x_1$	$+51{\cdot}8x_2$	$+28{\cdot}4x_3$	$+14{\cdot}1x_4$	$+5{\cdot}95x_5$
	$n=2$:				
	$6{\cdot}4\times\frac{4{\cdot}0}{8}=3{\cdot}2$	$4{\cdot}0\times\frac{4{\cdot}0}{8}=2{\cdot}0$	—	—	—
	$15{\cdot}1\times\frac{7{\cdot}7}{15}=7{\cdot}7$	$7{\cdot}7\times\frac{7{\cdot}7}{15}=4{\cdot}4$	$5{\cdot}2\times\frac{7{\cdot}7}{15}=2{\cdot}7$	—	—
	$27{\cdot}0\times\frac{15{\cdot}7}{27}=15{\cdot}7$	$15{\cdot}7\times\frac{15{\cdot}7}{27}=9{\cdot}1$	$8{\cdot}4\times\frac{15{\cdot}7}{21}=4{\cdot}9$	$5{\cdot}8\times\frac{15{\cdot}7}{27}=3{\cdot}38$	—
	$42{\cdot}2\times\frac{27{\cdot}4}{46}=25{\cdot}2$	$27{\cdot}4\times\frac{27{\cdot}4}{46}=16{\cdot}2$	$16{\cdot}1\times\frac{27{\cdot}4}{46}=9{\cdot}5$	$9{\cdot}0\times\frac{27{\cdot}4}{46}=5{\cdot}36$	$6{\cdot}5\times\frac{27{\cdot}4}{46}=3{\cdot}86$
Q_2	$=51{\cdot}8x_1$	$+31{\cdot}7x_2$	$+17{\cdot}1x_3$	$+8{\cdot}74x_4$	$+3{\cdot}86x_5$
	$n=3$:				
	$15{\cdot}1\times\frac{5{\cdot}2}{15}=5{\cdot}2$	$7{\cdot}7\times\frac{5{\cdot}2}{15}=2{\cdot}7$	$5{\cdot}2\times\frac{5{\cdot}2}{15}=1{\cdot}8$	—	—
	$27{\cdot}0\times\frac{8{\cdot}4}{27}=8{\cdot}4$	$15{\cdot}7\times\frac{8{\cdot}4}{27}=4{\cdot}9$	$8{\cdot}4\times\frac{8{\cdot}4}{27}=2{\cdot}6$	$5{\cdot}8\times\frac{8{\cdot}4}{27}=1{\cdot}80$	—
	$42{\cdot}2\times\frac{16{\cdot}1}{46}=14{\cdot}8$	$27{\cdot}4\times\frac{16{\cdot}1}{46}=9{\cdot}5$	$16{\cdot}1\times\frac{16{\cdot}1}{46}=5{\cdot}6$	$9{\cdot}0\times\frac{16{\cdot}1}{46}=3{\cdot}15$	$6{\cdot}5\times\frac{16{\cdot}1}{46}=2{\cdot}27$
Q_3	$=28{\cdot}4x_1$	$+17{\cdot}1x_2$	$+10{\cdot}0x_3$	$+4{\cdot}95x_4$	$+2{\cdot}27x_5$
	$n=4$:				
	$27{\cdot}0\times\frac{5{\cdot}8}{27}=5{\cdot}8$	$15{\cdot}7\times\frac{5{\cdot}8}{27}=3{\cdot}38$	$8{\cdot}4\times\frac{5{\cdot}8}{27}=1{\cdot}80$	$5{\cdot}8\times\frac{5{\cdot}8}{27}=1{\cdot}25$	—
	$42{\cdot}2\times\frac{9{\cdot}0}{46}=8{\cdot}3$	$27{\cdot}4\times\frac{9{\cdot}0}{46}=5{\cdot}36$	$16{\cdot}1\times\frac{9{\cdot}0}{46}=3{\cdot}15$	$9{\cdot}0\times\frac{9{\cdot}0}{46}=1{\cdot}76$	$6{\cdot}5\times\frac{9{\cdot}0}{46}=1{\cdot}27$
Q_4	$=14{\cdot}1x_1$	$+8{\cdot}74x_2$	$+4{\cdot}95x_3$	$+3{\cdot}01x_4$	$+1{\cdot}27x_5$
	$n=5$:				
	$42{\cdot}2\times\frac{6{\cdot}5}{46}=5{\cdot}95$	$27{\cdot}4\times\frac{6{\cdot}5}{46}=3{\cdot}86$	$16{\cdot}1\times\frac{6{\cdot}5}{46}=2{\cdot}27$	$9{\cdot}0\times\frac{6{\cdot}5}{46}=1{\cdot}27$	$6{\cdot}5\times\frac{6{\cdot}5}{46}=0{\cdot}92$
Q_5	$=5{\cdot}95x_1$	$+3{\cdot}86x_2$	$+2{\cdot}27x_3$	$+1{\cdot}27x_4$	$+0{\cdot}92x_5$

junctions of the strips. Assume that the edge member at the cut-out is large enough for its strain energy to be negligible.

The end load will build up at one "strip junction" from zero to $L(x_m - x_{m-1})$ over a length L.

The strain energy for one strip junction is thus

$$\int_0^L \frac{L^2(x_m - x_{m-1})^2\,dL}{2Elt'} = \frac{L^3}{6Elt'}(x_m - x_{m-1})^2$$

$$= 2{\cdot}38 \times 10^{-3}(x_m - x_{m-1})^2.$$

In the x_6 region, the procedure is rather different. End load in a strip of width $r\delta z$ varies from zero at one end to

$$x_6[L\sin(z+\delta z) - L\sin z] = x_6 L\cos z\,\delta z$$

(*see* Fig. 23) at the other end, giving the strain energy per elementary strip

$$\int_0^L \frac{(x_6 L\cos z\,\delta z)^2\,dL}{2Er\,\delta z\,t'} = \frac{L^3 x_6^2\cos^2 z\,\delta z}{6Ert'}.$$

The total strain energy from end load in the x_6 region is thus

$$\frac{L^3 x_6^2}{6Ert'}\int_0^{47^\circ}\cos^2 z\,dz = 6{\cdot}6\times10^{-4}x_6^2\left[\frac{z}{2} + \frac{\sin 2z}{4}\right]_0^{47^\circ}$$

$$= 4{\cdot}34\times10^{-4}x_6^2.$$

(The x_6 region has been taken as ending at 47° and the last 8° has been included with the $x_5 - x_6$ junction.)

The total additional strain energy due to skin and stringers is thus

$$4{\cdot}34\times10^{-4}x_6^2 + 2{\cdot}38\times10^{-3}[(0{\cdot}777x_6 - x_5)^2 + (x_5 - x_4)^2 + (x_4 - x_3)^2 \ldots + (x_2 - x_1)^2].$$

From this the increments of $\partial U/\partial x_1$, $\partial U/\partial x_2$, ... can now be calculated, but examination of Appendix III shows that what are really needed are the increments of

$$\frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_1}\ ,\ \frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_2}$$

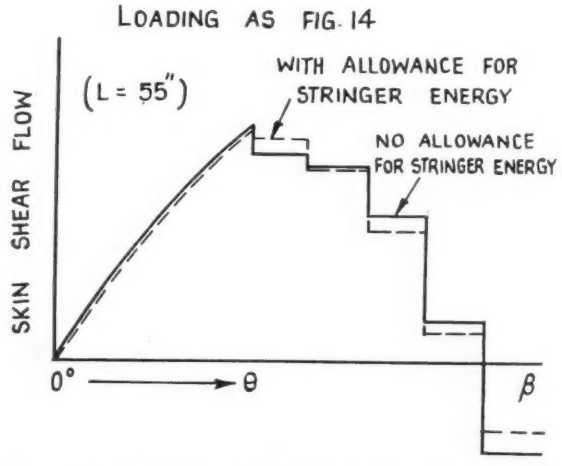

FIGURE 22. The effect of including stringer energy in the calculation of the shear flow in Fig. 21 (*See* Appendix IV).

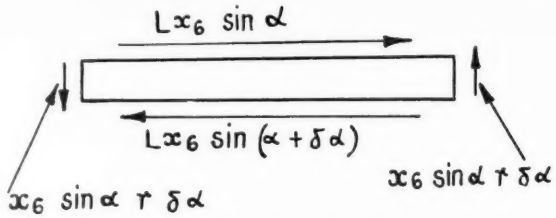

FIGURE 23. Shear loading of a strip of skin in "x_6" region (*See* Appendix IV).

et cetera, and accordingly these are calculated.

Increment of $\frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_1} = 1{\cdot}35\times10^3\times4{\cdot}76\times10^{-3}(x_1 - x_2)$

$= 6{\cdot}4(x_1 - x_2)$

" $\frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_2} = 12{\cdot}8(x_2) - 6{\cdot}4(x_1 + x_3)$

" $\frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_3}$, $\frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_4}$ similarly

" $\frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_5} = 12{\cdot}8x_5 - 6{\cdot}4x_4 - 4{\cdot}98x_6$

" $\frac{10^7}{(19{\cdot}5)^3}\frac{\partial U}{\partial x_6} = (1{\cdot}17 + 3{\cdot}86)x_6 - 5{\cdot}0x_5$

$= 5{\cdot}03x_6 - 5{\cdot}0x_5.$

The equations from Appendix III now become, with the direct additions of the terms just calculated,

$$\begin{aligned}
102{\cdot}6x_1 + 45{\cdot}4x_2 + 28{\cdot}4x_3 + 14{\cdot}1x_4 + 5{\cdot}95x_5 &= 1080\lambda\\
45{\cdot}4x_1 + 52{\cdot}0x_2 + 10{\cdot}7x_3 + 8{\cdot}74x_4 + 3{\cdot}86x_5 &= 1260\lambda\\
28{\cdot}4x_1 + 10{\cdot}7x_2 + 30{\cdot}33x_3 - 1{\cdot}45x_4 + 2{\cdot}77x_5 &= 1345\lambda\\
14{\cdot}1x_1 + 8{\cdot}74x_2 - 1{\cdot}45x_3 + 23{\cdot}34x_4 - 5{\cdot}13x_5 &= 1326\lambda\\
5{\cdot}95x_1 + 3{\cdot}86x_2 + 2{\cdot}27x_3 - 5{\cdot}13x_4 + 21{\cdot}25x_5 - 4{\cdot}98x_6 &= 1205\lambda\\
- 4{\cdot}98x_5 + 11{\cdot}68x_6 &= 1180\lambda\\
0{\cdot}799x_1 + 0{\cdot}934x_2 + 0{\cdot}996x_3 + 0{\cdot}982x_4 + 0{\cdot}891x_5 + 0{\cdot}88x_6 &= 25{\cdot}7.
\end{aligned}$$

The solution of these gives $x_1 = -2{\cdot}37$, $x_2 = 1{\cdot}0$, $x_3 = 4{\cdot}5$, $x_4 = 6{\cdot}7$, $x_5 = 7{\cdot}65$, $x_6 = 9{\cdot}9$, $\lambda = 0{\cdot}0652$. (All the numerical work is to slide rule accuracy only.)

The shears are plotted graphically in Fig. 22. The effect of including stringer energy has been a slight reduction in the maximum shear (*i.e.* from $x_6 = 10{\cdot}0$ to 9·9). For more flexible frames, a much greater effect would have been obtained. *See* Sections 14(*g*) and 19.

In the preceding, it has been tacitly assumed that after a length L the end load in the skin and stringers is redistributed in such a way that it becomes independent of x_1, x_2, and so on, and hence does not figure in the strain energy calculation. This redistribution could take place by bending the next frame out of its plane or by additional shear in the skins. The energy of these items has not been considered. It would constitute an additional complication, but its inclusion would be straightforward. It would increase the relief to the maximum shear.

APPENDIX V

SOLUTION OF A FRAME WITH A CUT-OUT AND INTERNAL REINFORCING

(*See* Figs. 24 to 26)

The centre line of the cross beam is defined by angle γ. The edge of the cut-out is defined by angle β. Other notation is as previously.

Bending moment in the beam $M_1 = 500\,x - M_0$.

Strain energy of half beam $= U_{\text{beam}}$

$$= \frac{1}{2EI_{\text{beam}}} \int_0^{\frac{b}{2}} (500\,x - M_0)^2\,dx.$$

Bending moment at α_m due to X_1, X_2 and M_0 is M'_m

where $M'_m = X_1 c_m - X_2 d_m + M_0$,

and $c_m = r \sin(\gamma - \alpha_m)$, $d_m = r' - r\cos(\gamma - \alpha_m)$.

$\dfrac{\partial M'_m}{\partial X_1} = c_m, \dfrac{\partial M'_m}{\partial X_2} = -d_m, \dfrac{\partial M'_m}{\partial M_0} = 1$ (provided that $\alpha_m > \gamma$).

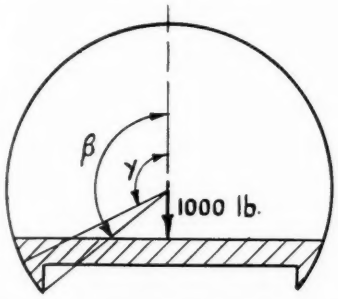

FIGURE 24. Cut frame reinforced with floor beam (*See* Appendix V).

The frame is divided into nine strips and I is taken as infinite for $m = 1$. Total strain energy of the half frame in bending is

$$\frac{l}{2E} \sum_{m=2}^{m=9} \frac{(M_m + M'_m)^2}{I_m}$$

Strain energy of the skin in shear is $\dfrac{Ll}{2tG} \sum_{n=1}^{n=9} x_n^2$.

Total strain energy U is given by

$$U = \frac{1}{2EI_{\text{beam}}} \int_0^{\frac{b}{2}} (500\,x - M_0)^2\,dx + \frac{Ll}{2tG} \sum_{n=1}^{n=9} x_n^2 + \frac{l}{2E} \sum_{m=2}^{m=9} \frac{(M_m + M'_m)^2}{I_m}$$

The equilibrium equations are

$$\sum_{n=1}^{n=9} x_n \sin \alpha_n = \frac{500}{l} \quad . \quad . \quad . \quad (a)$$

$$-X_1 \cos\gamma + X_2 \sin\gamma = 500 \quad . \quad . \quad (b)$$

Then, minimising by Lagrange's method,

$$\frac{\partial U}{\partial M_0} = \frac{1}{EI_{\text{beam}}} \int_0^{\frac{b}{2}} -(500\,x - M_0)\,dx + \frac{l}{E} \sum_{m=2}^{m=9} (M_m + M'_m) = 0$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

$$\frac{\partial U}{\partial X_1} = \frac{l}{E} \sum_{m=2}^{m=9} \frac{(M_m + M'_m)}{I_m} c_m = -\mu \cos\gamma \quad . \quad . \quad (d)$$

$$\frac{\partial U}{\partial X_2} = -\frac{l}{E} \sum_{m=2}^{m=9} \frac{(M_m + M'_m)}{I_m} d_m = \mu \sin\gamma \quad . \quad . \quad (e)$$

$$\frac{\partial U}{\partial x_n} = \frac{Ll}{tG} x_n + \frac{l}{E} \sum_{m=2}^{m=n} (M_m + M'_m)\, x_n = \lambda \sin x_n \quad . \quad (f)$$

Equations (*c*) to (*f*) yield 12 equations in x_1, $x_2 \ldots x_9$, X_1, X_2 and M_0 with terms in λ and μ and a constant on the right hand side. Solving in terms of λ and μ and the constant, and substituting these values in (*a*) and (*b*) gives numerical values for λ and μ and also for x_1, $x_2 \ldots x_9$, X_1, X_2, M_0.

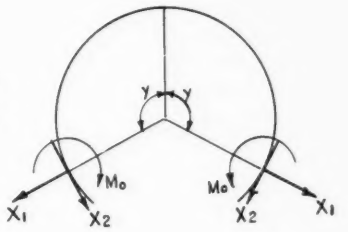

FIGURE 25. Frame of Fig. 24 showing forces applied to frame by floor beam (*See* Appendix V).

APPENDIX VI

(*i*) MINIMISING A FUNCTION OF SEVERAL VARIABLES USING LAGRANGE'S UNDETERMINED MULTIPLIERS

Let $U = f(x_1, x_2, x_3, \ldots x_n)$

and let $x_1, x_2, x_3, \ldots x_n$ be related by the expression

$$\phi(x_1, x_2, \ldots x_n) = 0.$$

When U has a stationary value (*i.e.* is a maximum or a minimum) it may be shown that:—

$$\frac{\partial U}{\partial x_1} = \lambda \frac{\partial \phi}{\partial x_1}$$

$$\frac{\partial U}{\partial x_2} = \lambda \frac{\partial \phi}{\partial x_2}$$

$$\ldots\ldots\ldots\ldots$$

$$\frac{\partial U}{\partial x_n} = \lambda \frac{\partial \phi}{\partial x_n}$$

where λ is an "unknown multiplier."

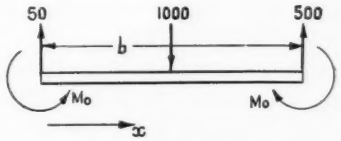

FIGURE 26. Forces applied to floor beam by frame (*See* Appendix V).

These n equations are sufficient to determine $x_1, x_2, \ldots x_n$ in terms of the unknown multiplier λ. Substitution of these values in $\phi(x_1, x_2, \ldots x_n)$ yields λ and hence $x_1, x_2, \ldots x_n$. If $(x_1, x_2, \ldots x_n)$ are related by two relations, *e.g.*

$$\phi(x_1, x_2, \ldots x_n) = 0$$

and $$\psi(x_1, x_2, \ldots x_n) = 0,$$

then it may be shown that at a stationary value

$$\frac{\partial U}{\partial x_1}=\lambda\frac{\partial \phi}{\partial x_1}+\mu\frac{\partial \psi}{\partial x_1}$$

$$\frac{\partial U}{\partial x_2}=\lambda\frac{\partial \phi}{\partial x_2}+\mu\frac{\partial \psi}{\partial x_2}\quad \text{etc.}$$

These n equations yield $x_1, x_2,\ldots x_n$ in terms of λ and μ. Substitution in $\phi(x_1, x_2,\ldots x_n)=0$ and $\psi(x_1, x_2,\ldots x_n)=0$ gives λ and ψ and hence $x_1, x_2,\ldots x_n$.

Example

Suppose $U=K_1x_1^2+K_2x_2^2+K_3x_3^2+K_4x_4^2$ (1)

and $a_1x_1+a_2x_2+a_3x_3+a_4x_4=e$ . . (2)

At a stationary value,

$$\frac{\partial U}{\partial x_1}=a\lambda=2K_1x_1 \qquad \frac{\partial U}{\partial x_2}=a_2\lambda=2K_2x_2$$

$$\frac{\partial U}{\partial x_3}=a_3\lambda=2K_3x_3 \qquad \frac{\partial U}{\partial x_4}=a_4\lambda=2K_4x_4.$$

Substitution in (2) yields

$$\frac{\lambda}{2}\left(\frac{a_1^2}{K_1}+\frac{a_2^2}{K_2}+\frac{a_3^2}{K_3}+\frac{a_4^2}{K_4}\right)=e,$$

giving finally

$$x_1=ae\Big/\left[K_1\left(\frac{a_1^2}{K_1}+\frac{a_2^2}{K_2}+\frac{a_3^2}{K_3}+\frac{a_4^2}{K_4}\right)\right]$$

$$x_2=ae\Big/\left[K_2\left(\frac{a_1^2}{K_1}+\frac{a_2^2}{K_2}+\frac{a_3^2}{K_3}+\frac{a_4^2}{K_4}\right)\right]\quad \text{etc.}$$

An example using both λ and μ will be found in Appendix V.

(*ii*) CROUT'S METHOD FOR THE SOLUTION OF EQUATIONS

This method can be applied to any equations but is specially suitable for arrays which are symmetrical about the leading diagonal. Details are too intricate to describe here, but can be found in Ref. 14. The method is best adapted for use with a computing machine.

(*iii*) GAUSS'S METHOD FOR THE SOLUTION OF EQUATIONS

(*See* Michielson, *Journal of the Aeronautical Sciences*, November 1949, p. 693)

This method applies to arrays of equations which are symmetrical about the leading diagonal. It represents a great saving of time and labour over the traditional methods.

The table below is self-explanatory except for the following:—(2, 3) means the coefficient of the third term in the second row. (This is normal matrix notation, the first suffix applying to rows and the second to columns). (3) means the complete third row of coefficients, and so on.

Equations for solution

(1) $3{\cdot}16\,x_1+4{\cdot}28\,x_2+5{\cdot}37\,x_3+2{\cdot}10\,x_4=\ 36{\cdot}32$

(2) $4{\cdot}28\,x_1+3{\cdot}46\,x_2+1{\cdot}07\,x_3-2{\cdot}92\,x_4=\ 2{\cdot}18$

(3) $5{\cdot}37\,x_1+1{\cdot}07\,x_2-3{\cdot}05\,x_3+1{\cdot}62\,x_4=\ 3{\cdot}41$

(4) $2{\cdot}10\,x_1-2{\cdot}92\,x_2+1{\cdot}62\,x_3-4{\cdot}85\,x_4=-19{\cdot}63.$

From (*j*) $x_4=\dfrac{-134{\cdot}770}{-33{\cdot}593}=4{\cdot}012$

$x_3=3{\cdot}012$ (from (*f*))

$x_2=2{\cdot}227$ (from (*c*))

$x_1=0{\cdot}692$ (from (*a*)).

Row (*a*) is apparently superfluous, being identical with

		x_1	x_2	x_3	x_4	*Right hand side*	*Check*
1	Coefficients of (1)	3·16	4·28	5·37	2·10	36·32	51·23
a	„ „ „	3·16	4·28	5·37	2·10	36·32	51·23
2	Coefficients of (2)	4·28	3·46	1·07	−2·92	2·18	8·07
b	$-\frac{(1,2)}{(1,1)}(a)$	−4·28	−5·797	−7·273	−2·844	−49·193	−69·387
c	$2+b$	0	−2·337	−6·203	−5·764	−47·013	−61·317
3	Coefficients of (3)	5·37	1·07	−3·05	1·62	3·41	8·42
d	$-\frac{(1,3)}{(1,1)}\times(a)$	−5·37	−7·273	−9·126	−3·569	−61·721	−87·059
e	$-\frac{(2,3)}{(2,2)}\times(c)$	0	6·203	16·464	15·299	124·785	162·751
f	$3+d+e$	0	0	4·288	13·350	66·474	84·112
4	Coefficients of (4)	2·10	−2·92	1·62	−4·85	−19·63	−23·68
g	$-\frac{(1,4)}{(1,1)}\times(a)$	−2·10	−2·844	−3·569	−1·396	−24·137	−34·046
h	$-\frac{(2,4)}{(2,2)}\times(c)$	0	5·764	15·299	14·216	115·953	151·233
i	$-\frac{(3,4)}{(3,3)}\times(f)$	0	0	−13·350	−41·563	−206·956	−261·869
j	$4+g+h+i$	0	0	0	−33·593	−134·770	−168·363

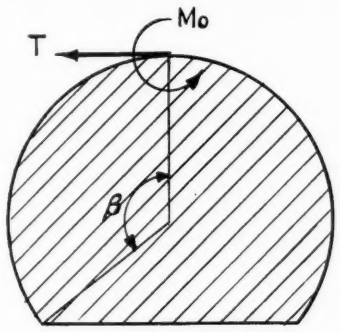

FIGURE 27. Incomplete bulkhead loaded with tangential or moment load on axis of symmetry (*See* Section 15(*e*)).

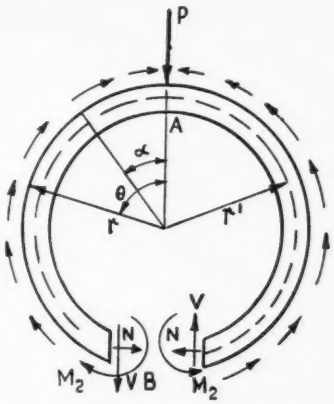

FIGURE 28. Radially loaded frame cut to show redundancies—skin line offset from median line of frame (*See* Appendix I).

Row 1. It is introduced, however, to preserve the symmetry of the notation. The check column is optional. For Rows 1, 2, 3, 4 it is obtained by adding the coefficients (including the constant term) in these rows. (If necessary the coefficients in any one column may be divided by 10 (or 100), to bring them to the same order as the other columns. This must then be done in every row). Operations carried out on the numbers in the rows are also carried out on the check column. From time to time a check addition is done to see that the sum of the coefficients in the row is still equal to the figure in the check column.

(*iv*) VON SEIDEL'S METHOD FOR THE SOLUTION OF EQUATIONS

This method is applicable only when the coefficients of the leading diagonal are considerably larger than the other coefficients. The leading diagonal terms are retained on the left hand side of the equation and the other terms carried across to the right hand side. Taking x_2, x_3, x_4 as zero, a value of x_1 is calculated from the first modified equation. Using this value, and taking x_3 and x_4 as zero, a value of x_2 is next calculated from the second modified equation. Similarly with x_3 and x_4, always using the latest available values of each unknown. Convergence is rapid if the given condition is satisfied.

$$\underline{5{\cdot}17}\ x_1 + 2{\cdot}14\ x_2 + 1{\cdot}85\ x_3 + 1{\cdot}62\ x_4 = 4{\cdot}18$$
$$2{\cdot}14\ x_1 + \underline{4{\cdot}23}\ x_2 + 1{\cdot}64\ x_3 + 1{\cdot}21\ x_4 = 3{\cdot}17$$
$$1{\cdot}85\ x_1 + 1{\cdot}64\ x_2 + \underline{3{\cdot}16}\ x_3 + 0{\cdot}93\ x_4 = 2{\cdot}43$$
$$1{\cdot}62\ x_1 + 1{\cdot}21\ x_2 + 0{\cdot}93\ x_3 + \underline{2{\cdot}94}\ x_4 = 1{\cdot}16.$$

$$x_1 = 0{\cdot}8085 - 0{\cdot}4139\ x_2 - 0{\cdot}3578\ x_3 - 0{\cdot}3133\ x_4$$
$$x_2 = 0{\cdot}7494 - 0{\cdot}5059\ x_1 - 0{\cdot}3977\ x_3 - 0{\cdot}2861\ x_4$$
$$x_3 = 0{\cdot}7690 - 0{\cdot}5854\ x_1 - 0{\cdot}5190\ x_2 - 0{\cdot}2943\ x_4$$
$$x_4 = 0{\cdot}3945 - 0{\cdot}5510\ x_1 - 0{\cdot}4116\ x_2 - 0{\cdot}3163\ x_3.$$

x_1	0·808511	0·696693	0·631610	0·611111	0·608176	0·610102	0·610561	0·610607
x_2	0·340375	0·416221	0·412692	0·400594	0·394156	0·391671	0·391721	0·391795
x_3	0·119000	0·212400	0·252083	0·263075	0·264369	0·263243	0·263043	0·262994
x_4	−0·228677	−0·227824	−0·203062	−0·190265	−0·186407	−0·186090	−0·186267	−0·186340.

A solution correct to one place of decimals was thus obtained in the third column, to two places in the fifth column and to three places in the seventh column. Had the approximate solution correct to one place been available, a solution to two places could thus have been obtained in about three columns, and to three places in about five columns.

(*v*) SOLUTION OF EQUATIONS BY RELAXATION METHODS

The method (due to Southwell) can be applied to equations of any type, not necessarily symmetrical about the leading diagonal. It is particularly adapted to obtaining a quick solution with the use of the slide rule, when an approximate solution is available. It can, however, be used *ab initio* if no approximate solution is available.

Using the same equations as in (*iii*) the solution *ab initio* is as follows. Choosing the variable with the largest coefficient in equation (1), and assuming the others are zero, write down its value. Applying this value to the other equations, write down the values of the left hand sides, and the residues obtained by subtracting these values from the right hand sides.

The largest residue is then selected and the corresponding equation examined to find the largest coefficient. A value is then assigned to the variable with this coefficient to make this residue smaller, and the other residues are correspondingly reduced (or increased).

It can be shown that it is advisable to over-compensate, *i.e.* to select a value for the variable greater than that required to liquidate the residue. It therefore pays throughout to work to the next highest simple number (*i.e.* the next highest integer, tenth, hundredth, and so on). This makes computation extremely easy.

The final values of the variables are obtained by adding together all the various values used. The example which follows will make this clear.

Where an approximate solution is available, the calculation starts with this value and the work is greatly shortened.

	$x_1=0{\cdot}9$		$x_2=0{\cdot}3$		$x_1=-0{\cdot}2$		$x_3=0{\cdot}3$	
x_1	4·653	−0·473	0·642	−1·115	−1·034	−0·081	0·555	−0·636
x_2	1·926	+1·244	1·269	−0·025	−0·428	+0·403	0·492	−0·089
x_3	1·665	+0·765	0·492	+0·273	−0·370	+0·643	0·948	−0·305
x_4	1·458	−0·298	0·363	−0·661	−0·324	−0·337	0·279	−0·616

	$x_4=-0{\cdot}3$		$x_2=0{\cdot}07$		$x_1=-0{\cdot}06$		$x_4=0{\cdot}10$	
x_1	−0·486	−0·150	0·150	−0·300	−0·310	+0·010	0·162	−0·152
x_2	−0·363	+0·274	0·296	−0·022	−0·128	+0·106	0·121	−0·015
x_3	−0·279	−0·026	0·115	−0·141	−0·111	−0·030	0·093	−0·123
x_4	−0·882	0·266	0·085	+0·181	−0·097	+0·278	0·294	−0·016

	$x_1=-0{\cdot}03$		$x_3=-0{\cdot}02$		$x_2=0{\cdot}020$		$x_3=-0{\cdot}010$	
x_1	−0·155	−0·003	−0·037	+0·034	0·043	−0·009	−0·019	0·010
x_2	−0·064	+0·049	−0·033	+0·082	0·085	−0·003	−0·016	0·013
x_3	−0·056	−0·067	−0·063	−0·004	0·033	−0·037	−0·032	−0·005
x_4	−0·049	+0·033	−0·019	+0·052	0·024	+0·028	−0·009	0·037

	$x_4=0{\cdot}01$		$x_3=-0{\cdot}005$	
x_1	0·016	−0·006	−0·009	0·003
x_2	0·012	+0·001	−0·008	0·009
x_3	0·009	−0·014	−0·016	0·002
x_4	0·029	−0·008	−0·005	−0·003

Final solution

$$\begin{aligned} x_1 &= 0{\cdot}9-0{\cdot}2-0{\cdot}06-0{\cdot}03 = 0{\cdot}61,\\ x_2 &= 0{\cdot}3+0{\cdot}07+0{\cdot}02 = 0{\cdot}39,\\ x_3 &= 0{\cdot}3-0{\cdot}02-0{\cdot}01-0{\cdot}005 = 0{\cdot}265,\\ x_4 &= -0{\cdot}3+0{\cdot}10+0{\cdot}01 = -0{\cdot}19, \end{aligned}$$

agreeing with the preceding solution.

APPENDIX VII

EFFECT OF STRINGER ENERGY ON THE SHEAR DISTRIBUTION AT A RIGID FRAME

(*See* Section 19)

From Appendix III, the strain energy in end load of skin and stringers (uniformly distributed round the frame) is

$$\frac{L^3}{6Elt'}\sum_{m=1}^{m=p}(x_m-x_{m-1})^2.$$

When this energy is included, equation (6) of Section 14 becomes, for a rigid frame

$$U=\frac{Ll}{2tG}\sum_{n=1}^{n=p}x_n^2+\frac{L^3}{6Elt'}\sum_{m=1}^{m=p}(x_m-x_{m-1})^2,$$

or in calculus notation

$$U=\frac{Lr}{2tG}\int_0^\beta z^2\,dz+\frac{L^3}{6Et'r}\int_0^\beta\left(\frac{dx}{dz}\right)^2dz,$$

with the condition from overall equilibrium (for a radial load P)

$$\frac{P}{2r}=\int_0^\beta x\sin z\,dz$$

(β being as in Fig. 27).

Moreover, if there is an end load carrying boom at β in addition to the uniformly distributed end load carrying material, a term $L^3x_\beta^2/(6EA_B)$ must be added to the strain energy, where A_B is the end load carrying area of the boom and x_β is the value of x at β.

Now x is a function of z, say $F(z)$. The problem is to find F when U has a stationary value. Using the methods of the calculus of variations (*see*, for instance, Jeffreys and Jeffreys, *Methods of Mathematical Physics*, p. 315 *et seq.*):—

$$\delta U=\left[\frac{\partial f}{\partial p}\delta x\right]_0^\beta+\int_0^\beta\left[\frac{\partial f}{\partial x}-\frac{d}{dz}\left(\frac{\partial f}{\partial p}\right)\right]\delta x\,dz+\frac{L^3x_\beta\delta x}{3EA_B}$$

where $p=\dfrac{dx}{dz}$, $f=K_1x^2+K_2p^2-\lambda x\sin z$,

$$K_1=\frac{Lr}{2tG},\quad K_2=\frac{L^3}{6Et'r},$$

and λ is undetermined. (Note: δx, δU represent small changes in U and x *for a fixed* z). Hence

$$\delta U=\left[2K_2p\delta x\right]_0^\beta+\int_0^\beta\left(2K_1x-\lambda\sin z-2K_2\frac{d^2x}{dz^2}\right)\delta x\,dz+\frac{L^3x_\beta\delta x}{3EA_B},$$

the last term being dropped when there is no boom. Now for a complete frame, $\beta=\pi$. Also x has stationary values at both ends ($z=0$ and $z=\pi$) and hence $\delta x=0$ at $z=0$ and $z=\pi$. (The fixed values are both zero by symmetry). For a frame with a cut-out, $\beta\neq\pi$ and two cases arise:—

(*a*) There is no end load carrying boom at β.

(*b*) There is an end load carrying boom at β in addition to the uniformly distributed end load carrying material, as already described.

In the first case x has fixed values (*i.e.* zero) at both ends ($\alpha=0$ and $\alpha=\beta$). Hence $\delta x=0$ at both ends. In the second case x has a fixed value (zero) at $\alpha=0$, but at $\alpha=\beta$, x is indeterminate. In this case, it readily follows that at $\alpha=\beta$,

$$2K_2 p+\frac{L^3 x_\beta}{3EA_B}=0,$$

i.e. that at $\alpha=\beta$,

$$\frac{dx}{d\alpha}=-\frac{L^3 x_\beta}{3EA_B\times 2K_2}=-\frac{t'r x_\beta}{A_B}.$$

In all three cases it also follows that

$$2K_1 x-\lambda\sin\alpha-2K_2\frac{d^2x}{d\alpha^2}=0,$$

i.e.

$$\frac{d^2x}{d\alpha^2}-x\frac{K_1}{K_2}=-\frac{\lambda\sin\alpha}{2}.$$

The general solution is,

$$x=A_1\sinh\alpha\sqrt{(3B)}+A_2\cosh\alpha\sqrt{(3B)}+\frac{\lambda}{2(K_1+K_2)}\sin\alpha,$$

where B is K_1/K_2 and is identical with the parameter B of Section 19. Applying the boundary conditions already discussed for the three types of frame considered:—

Complete frame

$$A_1=A_2=0 \text{ and } x \text{ varies as } \sin\alpha,$$

i.e. shear is sinusoidal.

Frame with cut-out, no boom at free edge

$$A_2=0,\quad A_1=\frac{-C\sin\beta}{\sinh\beta\sqrt{(3B)}},\quad \text{where } C=\frac{\lambda}{2(K_1+K_2)}$$

i.e.

$$x=C\left(\sin\alpha-\sin\beta\frac{\sinh\alpha\sqrt{(3B)}}{\sinh\beta\sqrt{(3B)}}\right) \text{ and at } \alpha=\beta,\ x=0.$$

Shear is clearly far from sinusoidal. This case, however, is unlikely to occur in practice.

Frame with cut-out, free edge reinforced with boom

$$A_2=0,\ A_1=\frac{-C(A_B\cos\beta+t'r\sin\beta)}{A_B\sqrt{(3B)}\cosh\beta\sqrt{(3B)}+t'r\sinh\beta\sqrt{(3B)}}.$$

i.e.

$$x=C\Big(\sin\alpha-\frac{A_B\cos\beta+t'r\sin\beta}{A_B\sqrt{(3B)}\cosh\beta\sqrt{(3B)}+t'r\sinh\beta\sqrt{(3B)}}\sinh\alpha\sqrt{(3B)}.\Big)$$

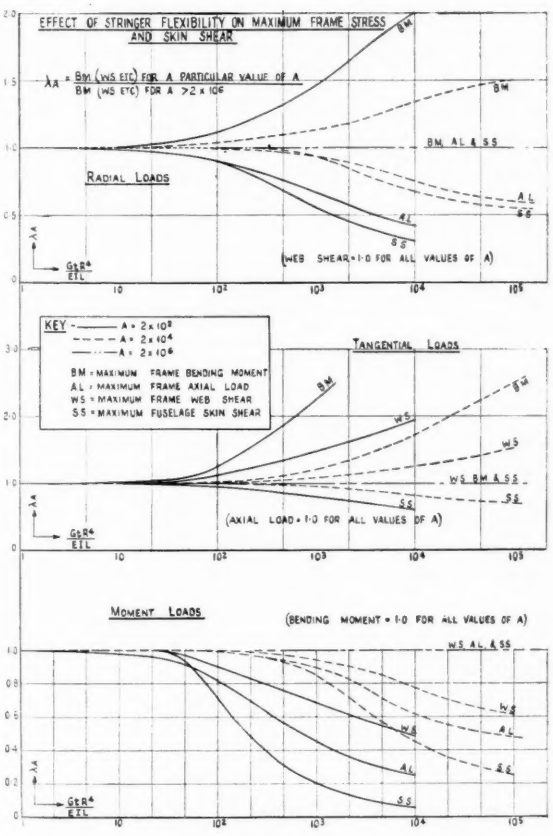
EFFECT OF STRINGER FLEXIBILITY ON MAXIMUM FRAME STRESS AND SKIN SHEAR
RADIAL LOADS
TANGENTIAL LOADS
MOMENT LOADS
KEY
BM = MAXIMUM FRAME BENDING MOMENT
AL = MAXIMUM FRAME AXIAL LOAD
WS = MAXIMUM FRAME WEB SHEAR
SS = MAXIMUM FUSELAGE SKIN SHEAR
(WEB SHEAR = 1·0 FOR ALL VALUES OF A)
(AXIAL LOAD = 1·0 FOR ALL VALUES OF A)
(BENDING MOMENT = 1·0 FOR ALL VALUES OF A)

FIGURE 29.

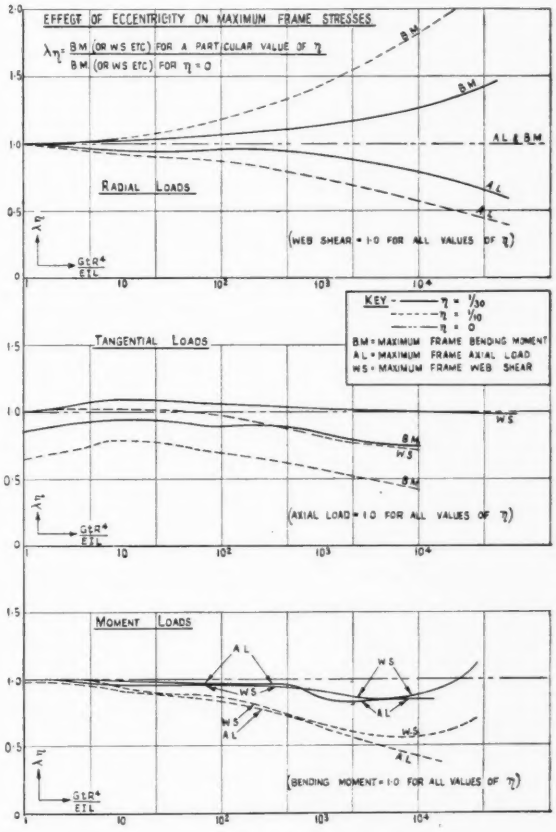
EFFECT OF ECCENTRICITY ON MAXIMUM FRAME STRESSES
RADIAL LOADS
TANGENTIAL LOADS
MOMENT LOADS
KEY
BM = MAXIMUM FRAME BENDING MOMENT
AL = MAXIMUM FRAME AXIAL LOAD
WS = MAXIMUM FRAME WEB SHEAR
(WEB SHEAR = 1·0 FOR ALL VALUES OF η)
(AXIAL LOAD = 1·0 FOR ALL VALUES OF η)
(BENDING MOMENT = 1·0 FOR ALL VALUES OF η)

FIGURE 30.

Since B is large (of the order of 100 upwards for practical frames), for all but small values of α,

$$x = C(\sin\alpha - \xi e^{-\sqrt{(3B)}(\beta-\alpha)}),$$

where ξ is $\dfrac{t'r\sin\beta + A_B\cos\beta}{\sqrt{(3B)}\,A_B + t'r}$.

Putting $A_B = \rho t'r$, gives $\xi < \dfrac{\sqrt{(1+\rho^2)}}{1+\rho\sqrt{(3B)}}$.

Taking $B = 100$, $\rho = 2$, $\xi < 0{\cdot}17$

Taking $B = 100$, $\rho = 3$, $\xi < 0{\cdot}065$

This gives for $\rho = 2$, $x = C(\sin\alpha - 0{\cdot}17)$ as $\alpha \longrightarrow \beta$

and for $\rho = 3$, $x = C(\sin\alpha - 0{\cdot}065)$ as $\alpha \longrightarrow \beta$.

The second term in the bracket becomes smaller as α becomes smaller. Hence, in general, provided that the boom area is reasonably large, the shear distribution will be approximately sinusoidal, and the deviation from sinusoidal can be neglected.

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TECHNICAL NOTES

The increasing flood of technical information makes it difficult to publish even abstracts of such information and it has become still more difficult to publish technical items of information which do not reach the status of full-length papers.

Under this general heading of TECHNICAL NOTES it is hoped that members of the Society, and indeed all those who read THE JOURNAL, *will help in contributing just those very items of useful information which normally might not see the light of the printed page.*

There will be no set form for these Notes. They may be illustrated or not. They can be short papers of a thousand or two thousand words; or abstracts and comments upon unpublished papers or upon published papers with a limited circulation; or notes of some interim results of research in hand and notes for further research; or in the form of a letter raising technical points or asking technical questions; or letters commenting on Notes already published.

These pages are intended to be a technical forum where members can argue and discuss; where they can inform and be informed; and pass on their own experience, or air their technical difficulties. Contributions will be published as soon as possible after they are received and will be eligible for JOURNAL *Premium Awards.*

No member need ever say that his particular subject does not get discussed in the pages of THE JOURNAL. *He can always raise it himself, for these pages are his pages.*

Three Notes are published this month. A treatment of vortices by potential theory, a consideration of the velocity components for swept-back wings at incidence, which gives an example of the errors introduced by the usual practice as compared with the exact method of resolution; and a theoretical discussion of minimum comfortable cruising speed. It is felt in some quarters that a minimum comfortable cruising speed for turbo-jet aircraft does not exist, and it will be interesting to have the comments and experiences of operators on Mr. Toms' Note.—THE EDITOR.

Resolution of Velocity for the Swept-Back Wing at Incidence

by

C. H. McKOEN, Ph.D., D.I.C.

(Sir W. G. Armstrong Whitworth Aircraft Co. Ltd.)

THE COMMONLY ACCEPTED practice of resolving a velocity V at incidence α into components $V \sin \Lambda$ parallel to the leading edge and $V \cos \Lambda$ normal to the leading edge at incidence $\alpha \sec \Lambda$ is only approximately true and the exact method of resolution is developed here.

In dealing with the aerodynamics of swept-back wings the free stream velocity may be resolved into a constant component parallel to the leading edge and a variable component normal to the leading edge. This is only possible if the constant percentage chord lines are all parallel to the leading edge, that is there is no taper, thickness taper or twist on the wing. This note gives the exact form and the small angle approximations of the velocity components and of the incidence of the component normal to the leading edge.

There are two cases to be considered, the swept-back wing with and without dihedral, at incidence. There is no effect with dihedral alone as the incident velocity remains in the plane of the wing and may be resolved parallel to and normal to the leading edge in the plane of the wing.

1. DEFINITION OF AIRCRAFT INCIDENCE

Aircraft incidence is applied by rotation about an axis perpendicular to the free stream direction, that is, parallel to *AB* (*see* Fig. 1). Incidence cannot be applied to a swept-back wing by rotation about the trailing edge, as this would imply breaking the wing along the plane of symmetry.

Received 19th January 1953

2. RESOLUTION OF VELOCITY FOR THE SWEPT-BACK WING WITHOUT DIHEDRAL

If α is the aircraft incidence and Λ is the angle of sweepback, then, using axes as in Fig. 1:—

Velocity V resolves into

(*i*) $V \cos \alpha$ parallel to the x-axis

(*ii*) $V \sin \alpha$ parallel to the z-axis.

$V \cos \alpha$ component resolves into

(*i*) $V \cos \alpha \sin \Lambda$ parallel to the leading edge

(*ii*) $V \cos \alpha \cos \Lambda$ normal to the leading edge.

Velocities $V \cos \alpha \cos \Lambda$ normal to the leading edge and $V \sin \alpha$ parallel to the z-axis compound into

$V_1 = V\sqrt{(\cos^2 \alpha \cos^2 \Lambda + \sin^2 \alpha)}$ normal to the leading edge at an angle of incidence β, where $\tan \beta = \tan \alpha \sec \Lambda$ (*see* Fig. 1).

Thus velocity V resolves into

(*i*) $V \cos \alpha \sin \Lambda$ parallel to the leading edge

(*ii*) $V\sqrt{(\cos^2 \alpha \cos^2 \Lambda + \sin^2 \alpha)}$ normal to the leading edge at incidence $\beta = \tan^{-1}(\tan \alpha \sec \Lambda)$.

For small angles of incidence this becomes

(*i*) $V \sin \Lambda$ parallel to the leading edge

(*ii*) $V \cos \Lambda$ normal to the leading edge at an incidence $\beta = \alpha \sec \Lambda$.

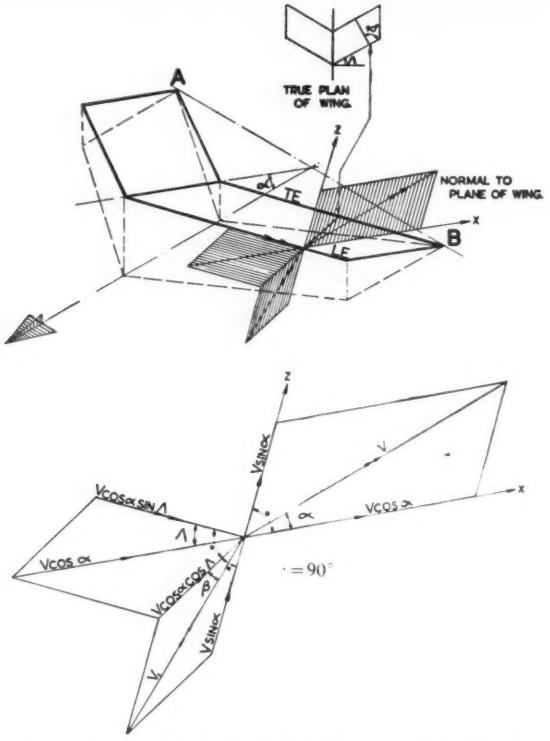

FIGURE 1. Resolution of velocity for swept wing without dihedral at incidence.

3. THE EFFECT OF DIHEDRAL ON THE PLANE OF THE WING

Dihedral is put on a swept-back wing by rotation about the line of symmetry. The chord lines in the downwind direction remain horizontal but any other line in the plane of the wing is not parallel to its original direction after dihedral has been put on the wing. We require to know the angle between the plane of the wing and the vertical plane through the leading edge for a swept-back wing with dihedral, at zero incidence. If this angle is θ, then in Fig. 2, angle $DEG = \theta$.

DE is the normal to the leading edge in the vertical plane.

GE is the normal to the leading edge in the plane of the wing.

γ is the angle of dihedral of the leading edge.

From the triangle GED,

$$GD^2 = GE^2 + ED^2 - 2GE\ ED\cos\theta$$

or $$FG^2 - (FG\sin\Lambda\sec\gamma)^2 = (FG\cos\Lambda)^2 +$$

$$(FG\sin\Lambda\tan\gamma)^2 - 2FG^2\sin\Lambda\cos\Lambda\tan\gamma\cos\theta$$

whence $$\cos\theta = \tan\Lambda\tan\gamma \quad . \quad . \quad . \quad (1)$$

4. RESOLUTION OF VELOCITY FOR SWEPT-BACK WING WITH DIHEDRAL

Using axes as in Fig. 3:—

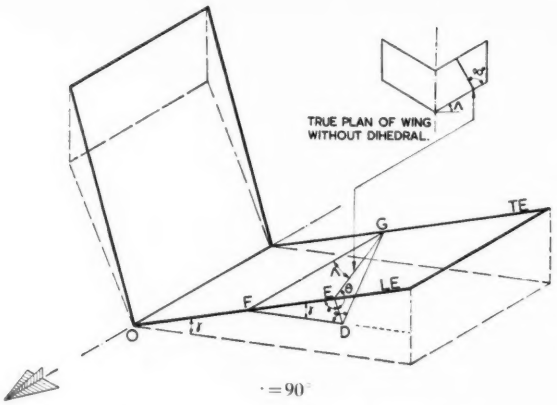

FIGURE 2. Swept wing with dihedral at zero incidence.

Velocity V at angle of incidence α resolves into

(*i*) $V\cos\alpha$ parallel to the x-axis
(*ii*) $V\sin\alpha$ parallel to the z-axis.

$V\cos\alpha$ parallel to the x-axis resolves into

(*i*) $V\cos\alpha\sin\Lambda$ parallel to the leading edge
(*ii*) $V\cos\alpha\cos\Lambda$ normal to the leading edge

} in the plane of the wing.

$V\sin\alpha$ parallel to the z-axis resolves into

(*i*) $V\sin\alpha\sin\gamma$ parallel to the leading edge
(*ii*) $V\sin\alpha\cos\gamma$ normal to the leading edge

} in the plane DEF of Fig. 2.

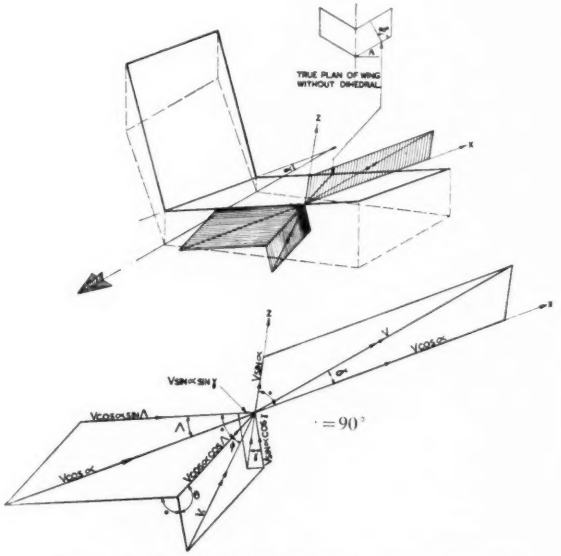

FIGURE 3. Resolution of velocity for swept wing with dihedral at incidence.

$V \cos \alpha \cos \Lambda$ normal to the leading edge in the plane of the wing and $V \sin \alpha \cos \gamma$ normal to the leading edge in the plane *DEF*, compound into

$$V_1 = V\sqrt{\{\cos^2\alpha\cos^2\Lambda + \sin^2\alpha\cos^2\gamma - 2\sin\alpha\cos\alpha\cos\Lambda\cos\gamma\cos\theta\}}$$

$$= V\sqrt{\{\cos^2\alpha\cos^2\Lambda + \sin^2\alpha\cos^2\gamma - 2\sin\alpha\cos\alpha\sin\Lambda\sin\gamma\}}$$

normal to the leading edge at an angle of incidence ϕ,

where $$\tan\phi = \frac{\sin\alpha\cos\gamma\sin\theta}{\cos\alpha\cos\Lambda - \sin\alpha\cos\gamma\cos\theta}.$$

Thus velocity V at incidence α resolves into

(*i*) $V(\cos\alpha\sin\Lambda + \sin\alpha\sin\gamma)$ parallel to the leading edge

(*ii*) V_1 normal to the leading edge at incidence ϕ.

For small angles of dihedral γ this reduces to

(*i*) $V(\cos\alpha\sin\Lambda + \gamma\sin\alpha)$ parallel to the leading edge

(*ii*) $V\sqrt{(\cos^2\alpha\cos^2\Lambda + \sin^2\alpha - 2\gamma\sin\alpha\cos\alpha\sin\Lambda)}$ at incidence ϕ,

where

$$\tan\phi = \tan\alpha\sec\Lambda + \gamma\tan\Lambda[\tan\alpha\sec\Lambda]^2.$$

5. NUMERICAL EXAMPLE

To illustrate the application of these formulae, the magnitude and incidence of the velocity component normal to the leading edge have been calculated for a wing with 45° sweepback with no dihedral and with 5° dihedral at various aircraft incidences. These values are given in Tables I and II compared to the values obtained from the approximate expressions $\cos\Lambda$ and $\alpha\sec\Lambda$.

TABLE I

MAGNITUDE OF VELOCITY COMPONENT

$\alpha°$	$\cos\Lambda$	V_1/V for $\gamma=0$	V_1/V for $\gamma=5°$
0	0·7071	0·7071	0·7071
5	0·7071	0·7098	0·7022
10	0·7071	0·7177	0·7027
15	0·7071	0·7304	0·7086
20	0·7071	0·7473	0·7197

TABLE II

INCIDENCE OF VELOCITY COMPONENT

$\alpha°$	$\alpha\sec\Lambda°$	$\beta°$	$\phi°$ for $\gamma=5°$
0	0	0	0
5	7·071	7·053	7·058
10	14·142	14·002	14·152
15	21·213	20·754	21·191
20	28·284	27·236	28·055

The Potential and Field of Flow (Induced Velocity) of a Plane Vortex

by

J. LOCKWOOD TAYLOR, D.Sc.

(Norges Tekniske Høgskole, Trondheim)

VORTICES are not usually treated by potential theory, but the latter is so well known and of such general application that there seem to be advantages in doing so.

Consider first the potential function

$$\phi = \tan^{-1}\left(\frac{zx}{yr}\right) \text{ where } r = \sqrt{(x^2+y^2+z^2)}$$

i.e. distance from *O*.

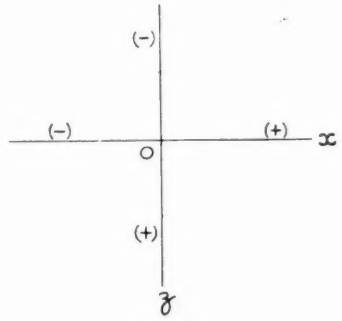

FIGURE 1.

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It can be shown by differentiation that this satisfies

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

as required for irrotational (perfect fluid) motion.

Also, along the x axis, $\pm x = r$,

$$\phi \longrightarrow \tan^{-1}\left(\frac{z}{y}\right), \; x \text{ positive}$$

$$\phi \longrightarrow -\tan^{-1}\left(\frac{z}{y}\right), \; x \text{ negative.}$$

These are recognised as the potential of a straight vortex line, with the sign of the vorticity changing at *O*.

Similarly along the z axis, $\pm z = r$

$$\phi \longrightarrow \tan^{-1}\left(\frac{x}{y}\right), \; z \text{ positive}$$

$$\phi \longrightarrow -\tan^{-1}\left(\frac{x}{y}\right), \; z \text{ negative}$$

i.e. there is also a vortex line along Oz, with change of sign at *O*.

The cruciform vortex represented by

$$\phi = \tan^{-1}\left(\frac{zx}{yr}\right)$$

can be converted into an H-vortex by taking

$$\phi = \tan^{-1}\left(\frac{zx+\delta x}{yr'}\right) - \tan^{-1}\left(\frac{zx}{yr}\right)$$

or $$\phi = \frac{\partial}{\partial x}\left\{\tan^{-1}\frac{zx}{yr}\right\}\delta x = \frac{zy}{r(x^2+y^2)}\delta x$$

(Actually the two vertical lines of the H have coalesced into a line of doublets).

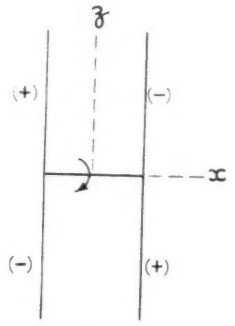

FIGURE 2.

When a series of such short vortices, length δx or δs, are combined by placing them end to end in a plane curve (or straight line), a vortex line of any form is obtained. The vertical line vortices now cancel out (provided the curve closes) and we have finally

$$\phi = \Sigma \frac{zy}{r(x^2+y^2)}\delta s$$

replacing δx by δs to show it is measured along the curve and remembering that x and y are now tangential and normal to the curve at each point, and z is perpendicular to the plane of the curve.

The induced velocity at a point in the plane of the curve

$$\left(\frac{\partial \phi}{\partial z}\right)_{(z=)} = \Sigma \frac{y}{r^3}\delta s, \qquad r^2 \text{ now} = x^2 + y^2$$

which is the well-known Biot-Savart law.

[Similarly for any other point if y is measured normal to δs and r is the distance from δs].

An alternative treatment, using the expression for

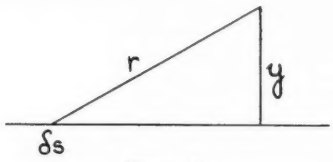

FIGURE 3.

the potential of a rectangular vortex derived in *The Aeronautical Quarterly* (November 1951), namely:

$$\phi = \tan^{-1}\left(\frac{x_1 y_1}{z r_1}\right) - \tan^{-1}\left(\frac{x_2 y_2}{z r_2}\right) + \\ + \tan^{-1}\left(\frac{x_3 y_3}{z r_3}\right) - \tan^{-1}\left(\frac{x_4 y_4}{z r_4}\right)$$

where the suffixes refer to co-ordinates measured from the corners of the rectangle in the plane of the vortex, *i.e.* now using cruciform vortices *in this plane*.

When the rectangle is small

$$\delta\phi = \delta S \frac{\partial^2}{\partial x \partial y}\left(\tan^{-1}\frac{xy}{zr}\right)$$

and in all

$$\phi = \left[\left[\tan^{-1}\frac{xy}{zr}\right]\right]$$

where the double brackets indicate insertion of limits corresponding with the boundary of the area, which is of course the vortex line.

This can be converted into a line integral round the vortex, but the expression previously given is probably more convenient.

Minimum Comfortable Cruising Speed

by

C. F. TOMS, B.Sc.(Eng.), A.F.R.Ae.S.

(Percival Aircraft Ltd.)

THE MINIMUM comfortable cruising speed of an aeroplane depends on the complexity of characteristics for the design in question, which may be broadly grouped under the headings of stability and control, drag and propulsion.

The store of knowledge relating to such speed is composed, in the main, from experience with aeroplanes powered by supercharged reciprocating engines, for which, at the low powers appropriate to the conditions considered, the brake horsepower and drag horsepower both increase with height at roughly the same rate (constant aeroplane weight and equivalent air speed being assumed). For a turbine-engined aeroplane the brake horsepower (or thrust) decreases with altitude and it might be supposed that this difference would result in a difference between the minimum comfortable speeds of similar aeroplanes driven by turbine engines (either jet or propeller) and by reciprocating engines. It appears, however, that a much more important difference arises from the change of thrust-speed characteristics as between jet and propeller-driven (whether turbine or reciprocating) aeroplanes.

The minimum comfortable cruising speed may be broadly defined as a mean speed which, together with

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the associated mean height, can be maintained under operational conditions, having regard to atmospheric and internally induced disturbances, with only moderate departures from the mean values and moderate times for recovery.

The analysis which follows is applicable to flight under normal or automatic control since it is concerned with the recovery characteristics of an aeroplane on the assumption that the governing factor is the manner of the dependence of thrust and drag on speed and height, stability considerations in the classical sense being set aside.

NOTATION

v_c rate of climb (ft./sec.)
v forward speed (true) (ft./sec.)
T thrust (lb.)
D total drag (lb.) $= T$
W aeroplane weight $\simeq$ lift, L
ρ air density (slugs/ft.^{3})
ρ_0 standard sea level air density
σ relative air density
D_i induced drag (lb.)
D_p non-induced drag (lb.)
C_{Dz} $\dfrac{D_p}{\frac{1}{2}\rho v^2 S}$
f $\dfrac{D_p}{\rho v^2}$
b wing span (ft.)
S wing area (ft.^{2})
B $\dfrac{2K}{\pi}\left(\dfrac{W}{b}\right)^2 = D_i \rho v^2$
P_0 engine power at sea level (h.p.)
T_0 thrust at sea level (lb.)
n exponent of σ for power- or thrust-altitude relationship
η propeller propulsive efficiency
h height (ft.)
z $\dfrac{v}{\eta}\dfrac{d\eta}{dv}$
m $\dfrac{v}{v_{md}}$
v_{md} true speed for minimum drag (ft./sec.)
K induced drag factor
A aspect ratio
g acceleration due to gravity (ft./sec.^{2})
v_s true stalling speed (ft./sec.)
t time (sec.)
η_A $\left(\dfrac{D}{L}\right)_{min}$

1. ANALYSIS

The rate of climb of an aeroplane is given by

$$v_c = \frac{T-D}{W} v \quad . \quad . \quad (1)$$

Hence
$$\frac{dv_c}{v_c} = \frac{d(T-D)}{T-D} + \frac{dv}{v}$$

or
$$dv_c = \frac{v}{W}(dT - dD) + v_c \frac{dv}{v} .$$

Considering initially level flight ($v_c = 0$),

$$\frac{dv_c}{dv} = \frac{v}{W}\left(\frac{dT}{dv} - \frac{dD}{dv}\right) .$$

Now
$$D = D_p + D_i = f\sigma\rho_0 v^2 + \frac{B}{\sigma\rho_0 v^2} .$$

Hence
$$\frac{dD}{dv} = \frac{dD_p}{dv} + \frac{dD_i}{dv}$$

and since
$$\frac{dD_p}{D_p} = \frac{d\sigma}{\sigma} + 2\frac{dv}{v}$$

and
$$\frac{dD_i}{D_i} = -\frac{d\sigma}{\sigma} - 2\frac{dv}{v} ,$$

$$\frac{dD}{dv} = (D_p - D_i)\left(\frac{2}{v} + \frac{1}{\sigma}\frac{d\sigma}{dv}\right)$$

$$= (D_p - D_i)\left(\frac{2}{v} + \frac{1}{\sigma}\frac{d\sigma}{dh}\frac{dh}{dv}\right) . \quad . \quad (2)$$

Considering the idealised cases of propeller-turbine engines (with negligible jet thrust) and pure jet engines (with thrust independent of speed) separately:—

Propeller-turbine

$$T = \frac{550 P_0 \sigma^n \eta}{v}$$

Hence
$$\frac{dT}{T} = n\frac{d\sigma}{\sigma} + \frac{d\eta}{\eta} - \frac{dv}{v}$$

or
$$\frac{dT}{dv} = D\left[\frac{n}{\sigma}\frac{d\sigma}{dh}\frac{dh}{dv} + \frac{z-1}{v}\right] . \quad . \quad (3)$$

Pure jet

$$T = T_0 \sigma^n$$

or
$$\frac{dT}{dv} = \frac{nD}{\sigma}\frac{d\sigma}{dh}\frac{dh}{dv} . \quad . \quad . \quad (4)$$

Now in the standard atmosphere, over a wide height range,

$$\frac{1}{\sigma}\frac{d\sigma}{dh} = -34 \times 10^{-6} . \quad . \quad (5)$$

Also
$$\frac{D_p}{D_i} = m^4 ,$$

leading to
$$D_p - D_i = \left(\frac{m^4 - 1}{m^4 + 1}\right) D . \quad . \quad (6)$$

Again
$$\frac{D}{W} \simeq \frac{D}{L} = \frac{1}{2}\left(\frac{D}{L}\right)_{min}\left(\frac{m^4 + 1}{m^2}\right) . \quad (7)$$

A combination and slight rearrangement of equations (1) to (7) gives the following expression for the derivative of v_c with respect to v:—

Propeller-turbine

$$\frac{dv_c}{dv} = -\frac{1}{2}\left(\frac{D}{L}\right)_{min}\left[\left(\frac{m^4+1}{m^2}\right)\left((1-z) + 34 \times 10^{-6} n v \frac{dh}{dv}\right) + \left(\frac{m^4-1}{m^2}\right)\left(2 - 34 \times 10^{-6} v \frac{dh}{dv}\right)\right] . \quad (8)$$

Pure jet

As equation (8) except for the absence of the term

$$(1-z) . \quad . \quad . \quad . \quad (9)$$

It is seen that a variable dh/dv remains and in connection therewith it is necessary to consider the kinds of disturbance which an aeroplane may suffer; here a

major difficulty arises. For the ideal conditions of still air the values of dh and dv in a small disturbance will be related in a simple manner. The possibility of small perturbations in height at constant air speed, and *vice versa*, in a disturbed atmosphere is obvious. A more remote, but not negligible, possibility is that of speed and height changes of the same sign occuring together. This latter possibility does not lend itself to any specification other than an arbitrary one, and will not be considered further here.

For still air conditions and small disturbances it is readily shown that

$$\frac{dh}{dv} = -\frac{v}{g} \quad . \quad . \quad . \quad (10)$$

and substitution in equation (8) gives, very approximately

$$\frac{dv_c}{dv} = -\frac{1}{2}\left(\frac{D}{L}\right)_{\min}\left[\left(\frac{m^4+1}{m^2}\right)\left((1-z)+\frac{nv^2}{10^6}\right)+ \right.$$
$$\left. +\left(\frac{m^4-1}{m^2}\right)\left(2-\frac{v^2}{10^6}\right)\right] \quad . \quad . \quad (11)$$

for propeller-turbine engines.

The value of dv_c/dv for given values of m, v, z and n is directly proportional to $(D/L)_{\min}$. Thus it is implied that, of two aeroplanes operating at a given speed the less aerodynamically efficient will be the more "speed-and height-stable." Since propeller-turbine and jet aeroplanes are respectively somewhat, and considerably, more efficient than those powered by reciprocating engines (aerodynamically)[1] it is implied that they are on this account the less stable in the present sense, other conditions being the same.

Since for a given aeroplane v varies with m, when evaluating equation (11) (and its counterpart for jet engines) mv_{md} is submitted for v, and various values of v_{md} can then be considered.

Figure 1 shows the variation of dv_c/dv with m from equation (11) for two values of $(D/L)_{\min}$, two values of v_{md} and of three values of n ($-0{\cdot}5$, $0{\cdot}5$ and $1{\cdot}0$; the first of these represents approximately the conditions for a supercharged reciprocating engine at constant r.p.m. and manifold pressure and the latter two the probable range of values for turbine engines). Also shown is dv_c/dv from equation (11) after putting $v^2=0$. It is seen that these terms are relatively unimportant and produce only small variations in the value of dv_c/dv.

To complete the presentation, equation (11) is plotted for two values of $(1-z)$ and the corresponding equation for jet engines for $v^2=0$ only.

Since putting $v^2=0$ introduces little error, and this amounts to putting $dh/dv=0$, it follows that the behaviour at low speeds is not significantly affected by the relationship between dh and dv in a disturbance, at least between the limits $dh/dv=0$ and $-v/g$, and hence equation (11) covers the case of independent disturbances to h and v.

2. CONCLUSIONS

On the assumption that the motion of an aircraft after disturbance at low speed conditions is governed only by the variations of thrust and drag with speed and altitude (*i.e.* that inertia and longitudinal stability in the classical sense are unimportant factors) it is shown that the minimum comfortable speeds for propeller-driven aeroplanes are substantially independent of the type of engines installed, but that, owing to the changed thrust-speed relationship, jet aeroplanes have slightly higher minimum comfortable speeds. It is also shown that the "aerodynamic efficiency" of an aeroplane, as defined by $(D/L)_{\min}$ has a noticeable effect on the speed in question; an increase in the "aerodynamic efficiency" (reduction in $(D/L)_{\min}$) increases the minimum speed.

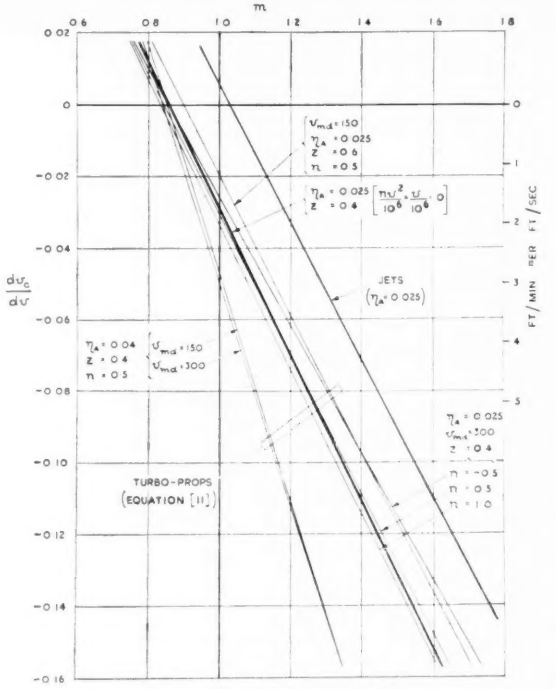

FIGURE 1.

If the negative value of dv_c/dv which represents acceptable low-speed characteristics for reciprocating-engined aeroplanes be taken as that corresponding to $m=1{\cdot}05$, then the corresponding values for a turbo-prop aeroplane are about $1{\cdot}10$ (allowing for a lower value of $(D/L)_{\min}$) and for a jet aeroplane about $0{\cdot}2$ greater.

It is advisable to check that the selected speed does not too closely approach the stalling speed; it can be shown that

$$\frac{v_s}{v_{md}} = \frac{1{\cdot}33}{(C_{L\max})^{\frac{1}{2}}}\left(\frac{A}{K}C_{Dz}\right)^{\frac{1}{4}}$$

It is appreciated that other considerations may contribute to a selection of a suitable value of m for flights of long duration; comments and information from manufacturers and operators of jet aeroplanes in particular would be welcomed.

REFERENCE

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Reviews

STRUCTURAL PRINCIPLES AND DATA. *Handbook of Aeronautics, No. 1. Pitman. 1953. Published under the authority of the Royal Aeronautical Society. Series Editors: C. G. Burge and J. L. Pritchard. Co-Editors, Structures: D. M. A. Leggett and M. Langley. Fourth Edition. 322 pp. Illustrated. 45s. net.*

The writing of a book is an affair of personal aim and personal effort; team work may be good for the schoolboy and sometimes, perhaps, for the scientist, but when it comes to writing a book, a team of authors is often in difficulties from the start. A realisation of this may be common ground in older fields of authorship, but it is currently viewed in aeronautical circles as slightly heretical or at least a little unsporting. Not least among aeronautical efforts at team writing was the first edition, in 1931, of the Handbook under review, and it was indeed the study of this at the time that first revealed the difficulties of the process to the present reviewer.

Then, as no doubt now, the editors did their best to enrol eminent and able authors and to instruct them in the aim of the work and their part in its achievement. But, nevertheless, the authors produced contributions that were an expression of themselves and their aeronautical interests without much reference to the rest, or else they never really put their heart in the matter and contented themselves with piling table upon table of rapidly dated data. The best contributions—those that have lasted in value—were those that took *no* notice of the others, a commonplace to anyone who cares to study scientific articles in early editions of the *Encyclopædia Britannica*.

This new volume, the first of several that are to constitute a 4th edition of the original work, opens with a list of four very well known and able editors. One wonders, is there to be a new and stronger effort to co-ordinate the authors? But no, the two parts of this book have as their authors men who cannot be expected to submit to a surfeit of co-ordination; let us defer judgment till we have studied each part.

Part I, with the title "Structural Airworthiness," is by W. Tye, the Chief Technical Officer of the Air Registration Board. As one would expect from its title, this part deals primarily with the loads that arise on aeroplane structures and the specification of values for these to ensure safe, yet efficient, design. The chapters deal in succession with manœuvring loads, gust loads, tail loads, aileron loads, undercarriage and engine mounting loads, crash loads and structural safety questions in general. The whole form an admirable, though condensed, statement of current airworthiness methods in this field, the best chapters, naturally enough, being those dealing with recent developments and the least satisfactory, perhaps, that on engine mountings, an almost static art.

Part II, with the title "Structural Analysis," is by J. H. Argyris, now of Imperial College, and P. C. Dunne, both of whom worked together until recent years on the Stressed-Skin Data Sheets of the Royal Aeronautical Society. This part deals mainly with aspects of structural analysis arising in modern aircraft structures and naturally assumes a background of engineering knowledge of roughly degree standard. Most of the chapters are concerned with basic information, with titles referring to stress-strain relations, beams and struts, buckling of plates, tension field shear panels, stress diffusion, and curved beams and rings, and only two deal specifically with larger aircraft components, entitled "Wing Stressing" and "Fuselages with Flexible Rings." The whole is followed by an extensive bibliography. The approach throughout is the critical scientific one typical of the authors, and no condensation of the matter is allowed to interfere with this.

Looking now at the two parts of this book in the light of our opening remarks, it is clear that, whatever the intention, the authors have *not* been unduly co-ordinated, and this the reviewer regards as, in the long run, a good thing. But as a result, and because of the different natures of the subjects and authors, the two parts will probably have different real uses. By its nature, no one can write a reference handbook on airworthiness that will long be used as such by designers; inevitably, their only safe handbook is that giving the official up-to-date requirements themselves. Part I must in the long run have educational rather than design uses, and for this it is admirable. Part II is also not really a section of a design handbook; it is a most interesting, critical text book in a specialist field and brings together for the first time much recent work in the Argyris-Dunne tradition, most of it, naturally enough, relating to fundamental rather than empirical knowledge.

The whole is well and accurately printed, and though the price is rather high, the publishers are to be congratulated on avoiding any competition with current American prices.—A.G.P.

STRUCTURE OF METALS. *C. S. Barrett. McGraw-Hill. 1952. 2nd edition. 661 pp. Diagrams. 72s. 6d. net.*

The appearance of a second edition of this book, first published over nine years ago, is evidence that although its scope is limited, it satisfies a definite need. The author, a physicist of the highest erudition, described his original work as being intended to serve both as a text and as a reference book. This description applies equally to the present edition, which, although nearly one hundred pages longer, is largely identical with its progenitor.

To the engineering metallurgist, structure has a wide significance. Such a man thinks of it as relating to the build-up of a casting, a forging or of an extrusion—that is to say, to a complete metallurgical entity. To Professor Barrett, however, the structure of metals is largely a question of the arrangement of a relatively small number of atoms in a unit often not so large as a single crystal. Since, however, in one of iron, having a volume of one cubic inch, there would be approximately $1{\cdot}4 \times 10^{24}$ atoms, one sees that the author's field is often a restricted one.

To a reader whose primary interest is limited to such

a microcosm, this carefully written and well reproduced work with its excellent descriptions of the applications of many X-ray, electron, atom and neutron techniques, will be of great value. On the other hand, to those whom one suspects of being more numerous, who look upon the structure of a single piece of metal as involving a heterogeneous conglomeration of allotriomorphic crystals, differing in size, orientation and chemical composition, containing physical discontinuities varying from Griffith cracks to substantial cavities and chemical ones, such as metallic and non-metallic inclusions, strangely retained gases and residual stresses, this scholarly work may be something of a disappointment.

By the selection of his title, Professor Barrett has let himself in for a certain amount of criticism—what he has done is analogous to that of an author whose book, entitled "The Structure of Houses" is, in fact, a learned essay on the making of bricks, with or without straw.—P. L. TEED.

AIRCRAFT INSTRUMENT DESIGN. *W. H. Coulthard. Pitmans, London 1952. 309 pp. Illustrated. Index. 40s. net.*

The technique of aircraft instrumentation is advancing with an ever-increasing tempo in an endeavour to keep pace with the aircraft themselves. It may even be suggested that increased endeavour is essential to keep in the race. Thus, nearly a quarter of a century after the appearance of "Stewart's Aircraft Instruments," a comparable volume is more than overdue. Mr. Coulthard is to be congratulated on his attempt to fill the void and it is no credit to British industry that both these books were left to the initiative of members of the Instruments Department of the R.A.E.

Mr. Coulthard has presented his work in three main parts dealing respectively with the instruments designed for use by the Pilot, the Navigator and the Flight Engineer; a fourth part, written by Mr. W. O. Broughton, deals with Automatic Flight treated as an extension of the classic theory of aeroplane stability.

The pattern followed generally throughout the book is to survey in turn each of the important quantities and establish the physical bases of its measurement, after which, in each case, the author illustrates the principles by descriptions of typical modern instruments.

It is perhaps a disadvantage of this microscopic approach that the broad principles underlying the successful design of aircraft instruments are never touched upon. Surely, in a book of this class entitled "Aircraft Instrument Design," the reader might expect some guidance on display such as has been provided in the "Naval Motion Study Unit Report No. 47." More generally still, one might expect some classification of displays according to the purpose served. Some pilot's instruments are essentially mis-alignment indicators in closed servo loops of which the pilot is expected to act as the servo mechanism. For too long the instrument designers have been laying a macedoine of information in front of the harassed pilot and patting themselves on the back every time they added another plum. However, during the past decade, the designers have been showing increasing awareness of the necessity of making their servo loops stable, instead of depending upon the skill of the pilot in emulating the stage performers who can balance half a dozen unstable systems simultaneously. The Sperry "Zero Reader" is a well known example of this modern trend, but the reader will search in vain for any mention of this instrument or for any mention of the principle on which it is based.

Although an interesting analysis of the errors developed by vertical gyros during turning flight is included and data relating to the most modern gyro-magnetic compass show that compass errors of over 20° can be caused by turning even at subsonic speeds, the point is not stressed that gravity-monitored instruments will have to improve very fast if they are to keep pace with future aircraft. In fairness to Mr. Coulthard it must be said that he may have been handicapped by considerations of security.

No such excuse can be offered for a general looseness of expression which is likely to cause either irritation or slovenly thinking, according to the attitude of the reader. Why should the word "radiation" be used to embrace the total loss of heat from a heated pitot head and what does "centrifugal acceleration" mean? The latter expression is used not to describe the behaviour of the components of an atom bomb but to account for the northerly turning error of a compass. Mention must be made of a palpably incorrect formula for the distance between two points on a tangent to the earth's surface because innocent students could waste much time making false calculations based thereon.

The indexing can only be described as lamentable. In spite of its shortcomings, this book contains much valuable material and deserves to be widely read.—F. W. MEREDITH.

Additions to the Library

Brennan, D. TIME ENOUGH TO LIVE. Secker & Warburg. 1953.

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Reports

AERODYNAMICS

Boundary Layer

Investigations into the effect of continuous suction on laminar boundary-layer flow under adverse pressure gradients. B. Thwaites. R. & M. 2514.

The principal problem considered in this paper is that of the flow along a surface through which fluid is being continuously withdrawn at a constant velocity, the velocity of the stream outside the boundary layer having a constant and negative gradient. When no suction is applied Howarth has solved the flow, and the point of separation is known. With continuous suction, the ordinarp methods of using the momentum equation, for example Pohlhausen's method, break down, and a new method is used in this paper to find the point of separation when suction of constant velocity is applied. Only partial success may be recorded in this problem, but this account of progress so far is now given in the hope that the problem may attract other workers. The momentum equation of boundary-layer flow is also used to deduce other types of flow in which the velocity of suction or the velocity outside the boundary-layer is not constant. These examples are inserted by virtue of the simplifications of method involved, by which the momentum equation may be integrated exactly; one of them is particularly applicable to the velocity distribution near the leading-edge of a thin aerofoil at high lift-coefficient, in which a small amount of suction is sufficient to prevent the stall.—(1.1).

The asymptotic theory of boundary-layer flow with suction. Three parts. E. J. Watson. R. & M. 2619.

The subject of this report is the steady two-dimensional flow of a boundary layer over a permeable surface through which the fluid is withdrawn at a known rate of suction. This rate of suction is assumed in accordance with the hypotheses of the boundary layer, to be small compared with the stream velocity. Part I deals with the similar solutions of the boundary-layer equations, Part II with an arbitrary pressure distribution but constant suction velocity, and Part III with the general problem. Attempts are made in both Parts I and II to find when separation occurs, but only rough estimates can be made as the series do not converge well. In Part II the theory is applied to the flow over a porous circular cylinder in a uniform stream, and also to the use of suction round the nose of an aerofoil to prevent stalling at high incidence.—(1.1).

Generalization of boundary-layer momentum-integral equations to three-dimensional flows including those of rotating system. A. Mager. N.A.C.A. Report 1067.

Boundary-layer equations for application in three-dimensional flows are developed. With the use of a fixed velocity profile and an empirical friction law an approximate solution in closed integral form is obtained for a generalised boundary-layer momentum-loss thickness and flow deflection at the wall in the turbulent case.—(1.1).

Survey of transition point measurements at the N.L.L., mainly for two-dimensional flow over a N.A.C.A. 0018 *profile. H. Wijker. N.L.L. Amsterdam, Report A.*1269.

This report contains some remarks on experiments in the determination of the transition point. Results following various techniques are compared. Transition points on a N.A.C.A. 0018 profile are given for an angle of incidence between $-6°$ and $+10°$ and Reynolds numbers between $0{\cdot}4 \times 10^6$ and $2{\cdot}9 \times 10^6$.—(1.1).

Some experimental investigations on the influence of wall boundary layers upon wind tunnel measurements at high subsonic speeds. E. G. M. Petersohn. F.F.A. Sweden. Report 44.

Pressure distribution measurements and drag determination by means of balance measurements have been carried out for a number of models at high subsonic velocity in wind tunnels, where the boundary layer of the walls has been varied. The phenomenon described should be of a certain importance from the point of view of wind tunnel technique, since it is possible to increase the choking velocity for a given model by means of thickening the boundary layer.—(1.1 × 1.2).

Approximate calculation of the influence of wall boundary layers upon the blockage interference in a high speed wind tunnel. S. B. Berndt. F.F.A. Sweden. Report 45.

The wall boundary layers in a subsonic wind tunnel work to reduce the choking effect of a model. A simple approach to a theoretical explanation is used. It involves neglecting viscosity, linearisation of the equations of motion and substitution of wall layers of constant velocity for the boundary layers. Solutions are given for plane and axisymmetric flow. Results of numerical application to the case of a parabolic arc profile in plane flow confirm experimental results and indicate strong boundary layer effects to occur at Mach numbers close to one.—(1.1 × 1.2).

*Tests on four aerofoil cascades: Part II. Boundary layer characteristics. F. G. Blight and W. Howard. A.R.L. Australia, Report E.*75.

Tests on four cascades, described in Part I of this report, indicated the presence of a boundary layer separation on the blade upper surfaces. The nature of this separation has been investigated using oil film, surface pitot and hot wire methods, the most consistent and satisfactory results being obtained with the oil film.—(1.1 × 1.5).

Some investigations on thin nose-suction aerofoils, Parts I and II. J. Williams. R. & M. 2693.

The stalling properties of some thin nose-suction aerofoils already tested have been examined, and further theoretical investigations have been carried out on thin aerofoils specially designed to give high lift with nose-slot suction. In Part I, the experimental results from stalling tests on thin nose-suction aerofoils are compared and the design features of the tested aerofoils are analysed. Part II describes a theoretical exploration of possible thin nose-slot aerofoils specially designed to have an abrupt fall in velocity where suction is to be applied on the upper surface of the nose. The theoretical formulæ required for the calculation of the velocity distribution and lift of an aerofoil with a sink on its surface, and some detailed notes on the design of the Glauert nose-slot aerofoil, are given in the Appendices.—(1.1 × 1.10).

Compressible Flow

See also Reseach

Experiments on transonic flow around wedges. G. P. Wood. N.A.C.A. Technical Note 2829.

Several aspects of transonic flow around the forward portions of wedge profiles were studied by means of interferometry. Measurements were made of the two kinds of flow pattern that occur at the leading edge of a wedge at an angle of attack. The growth of the supersonic region at a sharp convex corner formed by two flat surfaces was observed. The pressure-drag coefficients of a wedge of 14·5° semi-angle were measured at Mach numbers of 0·768, 0·819, and 0·854, and were shown to be consistent with those of wedges of smaller angle when plotted according to the transonic similarity law. Conditions at the bases of the shock waves that interacted with boundary layers on the wedge were measured. The method of characteristics was used to calculate the flow behind an experimentally determined sonic line, and the calculated flow field was compared with the measured flow field. The accuracy in the location of the sonic line necessary to give correctly the pressure distribution on the surface behind it was determined.—(1.2).

Laminar boundary layer on cone in supersonic flow at large angle of attack. F. K. Moore. N.A.C.A. Technical Note 2844.

The laminar boundary-layer flow about a cone at large angles of attack to a supersonic stream has been analysed

NOTE:—*The figures in parenthesis at the end of each Summary are for office use only.*

in the plane of symmetry. At the bottom of the cone, velocity profiles were obtained showing the expected tendency of the boundary layer to become thinner on the under side of the cone as the angle of attack is increased. At the top of the cone, the analyses failed to yield unique solutions, except for small angle of attack. Beyond a certain critical angle of attack, boundary-layer flow does not exist in the plane of symmetry, thus indicating separation.—(1.2 × 1.1.).

Shock tube theory and applications. J. Lukasiewicz. N. A. E. Canada, Report 15.

The theory of shock tube flow is presented in a form convenient for aerodynamic design of shock tubes. Tables and graphs of the required functions, covering the whole range of values of the specific heat ratio, are included. Application of shock tubes to investigation of compressible flow is examined. In particular, the use of shock tubes as sub- and supersonic tunnels, as aeroballistic ranges (including hypersonic speeds) and for investigation of propagation of continuous waves, shocks (including variable specific heat and relaxation time effects) and interactions is considered.—(1.2 × 31.2).

Fluid Dynamics

Corrections for drag, lift, and moment of an axially symmetrical body placed in a supersonic tunnel having a two-dimensional pressure gradient. I. J. Kolodner, F. Reiche and H. F. Ludloff. N.A.C.A. Technical Note 2837.

Corrections for drag, lift, and moment are derived for an axially symmetrical body placed in a supersonic tunnel having a two-dimensional pressure gradient.—(1.4).

Low aspect ratio wings with small thickness at zero lift in subsonic and supersonic flow. F. Keune. K.T.H. Sweden. Aero T.N. 21.

The flow considered can be described by two terms: a cross-sectional flow orthogonal to the direction of flight and a spatial influence. The first term is a two-dimensional flow at any distance from the leading edge of the wing, corresponding to Jones's theory. The second term depends on the variation of the cross-sectional area in chordwise direction, analogous to the flow around bodies of revolution.—(1.4).

Internal Flow

Flow surfaces in rotating axial-flow passages. J. D. Stanitz and G. O. Ellis. N.A.C.A. Technical Note 2834.

In order to investigate the deviation of flow surfaces from their assumed orientation in the usual type of two-dimensional solution, three-dimensional, incompressible, non-viscous, absolute irrotational fluid motion is determined for flow through rotating axial-flow passages bounded by straight blades of finite spacing and infinite axial length lying on meridional planes.—(1.5).

Effect of changing passage configuration on internal-flow characteristics of a 48*-inch centrifugal compressor. II—change in hub shape. J. Mizisin and D. J. Michel. N.A.C.A. Technical Note* 2835.

The passage contour of a 48-inch centrifugal compressor was modified by changing the shape of the hub to control the deceleration rates along the blade surfaces in order to improve the internal efficiency of the impeller. A comparison of internal-flow characteristics at design flow rate was made with the original impeller and with a modified-blade impeller that had the same area variation in the passage. In addition, flow characteristics of the modified-hub impeller over a flow range from maximum flow to near surge at a corrected tip speed of 700 feet per second are presented.—(1.5).

Étude générale de l'écoulement d'un gaz a travers une tuyère quelconque et du passage par la vitesse du son. M. Serruys. Publications Scientifiques et Techniques du Ministère de L'Air, France, No. 272.—(1.5).

Die Verteilung der Durchflussmenge in einem ebenen Verzweigungssystem. H. Reichardt und W. Tollmien. Max-Planck-Institut, Göttingen, Mitteilungen Nr. 7.—(1.5).

Stability and Control

*The static longitudinal stability and control of an aeroplane as affected by the compressibility of the air. J. Buhrman and C. M. Kalkman. N.L.L. Amsterdam. Report V.*1625.

A computation method is given for the effect of the compressibility of the air on the static longitudinal stability and control characteristics of an aeroplane. The derived formulæ are based on the Prandtl rule. Their applicability is limited to Mach numbers below the critical Mach number since the linear theory holds no longer beyond this limit. In order to illustrate the magnitude of the influence of the Mach number on static and manœuvre margins and trim changes the results of a numerical application of the formulæ are included in the report.—(1.8).

Wings and Aerofoils

The scope and accuracy of vortex lattice theory. V. M. Falkner. R. & M. 2740.

The report gives an outline of the development of the principles on which potential problems in lifting-plane theory are solved by the use of a vortex lattice for the purpose of computing downwash. The conditions of convergence necessary for an accurate solution are defined, and the main purpose of the report is to show that those connected with the lattice have been, or can easily be, satisfied.—(1.10).

Langley full-scale-tunnel investigation of the maximum-lift and stalling characteristics of a trapezoidal wing of aspect ratio 4 *with circular-arc airfoil sections. R. H. Lange. N.A.C.A. Technical Note* 2823.

Results are given of an investigation at high Reynolds numbers and low Mach numbers to determine the maximum-lift and stalling characteristics of a trapezoidal wing of aspect ratio 4 with 10 per cent-thick, circular-arc aerofoil sections. The tests included measurements of the lift, the drag, and the pitching-moment coefficients of the basic wing and of the wing with 0·20-chord droop-nose and trailing-edge flaps deflected both alone and in various combinations. Scale effects were investigated at Reynolds number ranging from $3{\cdot}27 \times 10^6$ to $7{\cdot}67 \times 10^6$.—(1.10).

Effects of independent variations of Mach number and Reynolds number on the maximum lift coefficients of four N.A.C.A. 6*-series airfoil sections. S. F. Racisz. N.A.C.A. Technical Note* 2824.

The N.A.C.A. 65-006, 64-009, 64-210, and 64_2-215 aerofoil sections were tested in the Langley low-turbulence pressure tunnel to determine the effects of independent variations of Mach number and Reynolds number on the maximum-lift characteristics for the smooth and rough conditions.—(1.10).

Effect of a finite trailing-edge thickness on the drag of rectangular and delta wings at supersonic speeds. E. B. Klunker and C. Rennemann, Jr. N.A.C.A. Technical Note 2828.

The effect of a finite trailing-edge thickness on the pressure drag of rectangular and delta wings with truncated diamond-shaped aerofoil sections with a given thickness ratio is studied for supersonic Mach numbers, linearised theory being used to evaluate the surface pressures. In order to facilitate comparison with wings having sharp trailing edges, the position of maximum thickness and base height are determined for least pressure drag as functions of a base-pressure parameter. Comparison is then made between the drag of these wings and similar wings with a sharp trailing edge for various aspect ratios and thickness ratios as a function of stream Mach number. Calculations of the span-wise distribution of drag are included to compare further the effect of a base on the drag for different aspect ratios.—(1.10).

Span load distributions resulting from constant angle of attack, steady rolling velocity, steady pitching velocity, and constant vertical acceleration for tapered sweptback wings with streamwise tips. Subsonic leading edges and supersonic trailing edges. Margery E. Hannah and K. Margolis. N.A.C.A. Technical Note 2831.

On the basis of the linearised supersonic-flow theory the theoretical spanwise distributions of circulation (which are proportional to the span load distribution) resulting from constant angle of attack, steady rolling velocity, steady pitching velocity, and constant vertical acceleration were

calculated for a series of thin, swept-back, tapered wings with streamwise tips, subsonic leading edges, and supersonic trailing edges. The results of the analysis are presented as a series of design charts. Some illustrative variations of the spanwise distributions of circulation with aspect ratio, taper ratio, Mach number, leading-edge sweepback, and axis-of-pitch location are also included.—(1.10).

The flow over two-dimensional aerofoil at incidence in the transonic speed range. T. R. Gullstrand. K.T.H. Sweden. Aero T.N. 27.

The transonic integral equation, due to K. Oswatitsch, holds only for symmetrical aerofoils without incidence. With the help of Green's theorem, similar integral equations are in this report developed for aerofoils at incidence. If there are normal shock waves at the aerofoil it cannot be assumed that the additional velocity is antisymmetric because the shock waves move in different directions on the upper and lower surfaces of the aerofoil with change in angle of attack. If it is assumed, however, that the additional velocity is antisymmetric, except in that part of the flow where the shock waves are, approximate equations can be deduced, which can be solved in a way similar to that used for the other equations. As a result one also gets the extent of the shock waves movement with an angle of attack. As an example of the method, additional velocity distributions are calculated for some Mach numbers for a 6 per cent thick symmetrical parabolic arc aerofoil. The lift curve slope as function of the free stream Mach number is also calculated.—(1.10).

Untersuchungen an einem symmetrischen Tragflügel mit spaltlos angeschlossenem Ruder bei raschen Aenderungen des Ruderausschlags (ebene Strömung). H. Drescher. Max-Planck-Institut, Göttingen, Mitteilungen Nr. 6.—(1.10).

The flow over symmetrical aerofoils without incidence at sonic speed. T. R. Gullstrand. K.T.H. Sweden. Aero T.N. 24.

To treat transonic flow it is necessary to take into account non-linear terms in the flow equations, which are then very difficult to solve. If Green's theorem is applied, the differential equations can be changed to a non-linear integral equation. The value of the integrals in the equation can be calculated approximately if appropriate assumptions are made for the unknown velocity distribution in the region of integration. The resulting equation for the velocity distribution on the aerofoil is then solved by an iteration process. In an earlier report by the author the velocity distribution on some symmetrical aerofoils at zero lift was calculated in the lower transonic range. In the present report the flow over some symmetrical aerofoils at zero lift is calculated for sonic speed. The solutions are in agreement with available experimental results.—(1.10 × 1.2).

A theoretical discussion of some properties of transonic flow over two-dimensional symmetrical aerofoils at zero lift with a simple method to estimate the flow properties. T. R. Gullstrand. K.T.H. Sweden. Aero T.N. 25.

With the help of the transonic integral equation it is shown that in a region on both sides of sonic velocity the Mach number distribution on a symmetrical aerofoil without lift is independent of the free stream Mach number. The transonic similarity law is discussed and it is shown that in predicting the position of the shock wave on the aerofoil the law holds only for very thin aerofoils. When there are no shocks on the aerofoil the law holds to much larger thickness ratios.—(1.10 × 1.2).

AEROELASTICITY

Influence of tuned dampers on flexure-aileron flutter. Three parts. R. A. Frazer et al. R. & M. 2559.

In Part I a general theory is developed for the investigation of the influence of damping devices on various types on flexure-aileron flutter. The numerical applications refer to a large transport aircraft, and they are restricted to the case of a mass-balanced aileron-carried damper. Part II supplements Part I and gives results for a partly balanced and for a completely balanced aileron-damper system, and Part III describes an experimental investigation into the effect on flexure-aileron flutter of a tuned damping device attached to the aileron.—(2).

AIRCRAFT

See FLIGHT TESTING

AIRCRAFT DESIGN

The aerodynamic development of a delta type glider. V. J. Smith and E. W. Chambers. A.R.L. Australia, Aerodynamics Note 108.

This note covers field work and wind tunnel tests on several 25 and 60 inch span models to provide a stable practical glider for use in meteorological research. Various techniques employed are explained and a series of flight tests is described.—(4.1).

AIRCRAFT OPERATION

*Ditching tests on a 1/8-size model of the Fokker S-13 crew trainer. J. F. Hengeveld. N.L.L. Amsterdam, Report A.*1238.

A dynamically similar model of the Fokker S-13 (1/8 size) was subjected to a number of ditching tests at the N.L.L. open air ditching station. Conclusions were drawn on the effect of the attitude of the plane, on the character of the run, and on the influence of such minor variables as flaps, propellers, landing gear, etc.—(5.4 × 31.1).

FLIGHT TESTING

Harvard 4—flight trials. C.E.P.E., Canada, Report 988.

This report was prepared to give handling and performance data on Harvard 4 aircraft equipped with a propeller spinner, an auxiliary fuel tank and a large panel canopy. The handling characteristics were fully investigated but no effort was made to reduce any of the performance figures to standard conditions.—(13.1 × 3.11).

HYDRODYNAMICS

Motion of a cylinder under the surface of a heavy fluid. L. N. Sretensky. N.A.C.A. Technical Memorandum 1335.

General equations are derived for the approximate solution of the flow of a heavy infinite depth about a submerged circular cylinder where the centre of the cylinder is submerged 1·5 diameter or greater. Then by replacing the cylinder by a dipole of a certain strength an equation is derived for the wave resistance of a circular cylinder moving in a fluid of finite depth.—(17.1).

Tank tests on a jet-propelled boat-seaplane fighter (Saunders-Roe E6/44). G. L. Fletcher. R. & M. 2718.

This report describes investigations into porpoising stability, water resistance, and seaworthiness of the hull design of the E6/44.—(17.2).

A theoretical and experimental investigation of the effects of yaw on pressures, forces, and moments during seaplane landings and planing. R. F. Smiley. N.A.C.A. Technical Note 2817.

A theory for the side force, rolling moment, yawing moment, and pressure distribution during the yawed landings and planing of seaplanes was developed. For the special case of the straight-sided wedge without chine immersion, the results of the theoretical analysis are presented in the form of generalised curves covering all step landing conditions. Experimental impact and planing data are presented for a prismatic wedge having an angle of dead rise of 22·5° and are shown to be in reasonable agreement with the theoretical predictions.—(17.2).

MATERIALS

Sandwich construction and core materials. Part VI. Three sections. W. J. Pullen, et al. R. & M. 2687.

A range of struts each consisting of "Balsolite" filler sandwiched between two faces of 1 in. thick birch plywood was tested in order to assess the efficiency of Balsolite 16 as a stabiliser in sandwich structures. It is concluded that this material compares favourably with other low density materials when used as a stabiliser. Modification of the material, namely the use of transverse and longitudinal tubes alternately, does not appear to be beneficial.—(21.5 × 33.2.4).

MECHANICAL ENGINEERING

Experimental investigation of eccentricity ratio, friction, and oil flow of short journal bearings. G. B. DuBois and F. W. Ocvirk. N.A.C.A. Technical Note 2809.

An experimental investigation was conducted to obtain performance data on bearings of length-diameter ratios of 1, 1/2, and 1/4 for comparison with theoretical curves. A 1·375-inch-diameter bearing was tested at speeds up to 6,000 r.p.m. and with unit loads from 0 to 900 lb./in.^{2}. A method of plotting oil flow data as a single line is shown. Methods are also discussed for approximating a maximum bearing temperature and evaluating the effect of deflection or misalignment on the eccentricity ratio at the ends of the bearings.—(23.1).

Investigation of 75-millimeter-bore deep-groove ball bearings under radial load at high speeds. 1—Oil-flow studies. Z. M. Nemeth, E. F. Macks and W. J. Anderson. N.A.C.A. Technical Note 2841.

At an oil inlet temperature of 100°F. two methods of bearing lubrication, single jet and puddling, were investigated. In this investigation, 75-millimeter-bore ball bearings were studied over a range of D.N. values (bore times speed) from $0{\cdot}3 \times 10^5$ to $1{\cdot}2 \times 10^5$, radial loads from 7 to 1,113 lb., and oil flows from 2 to 8 lb./min. The effects of the operating variables on the oil flow through the bearing and upon the bearing outer-and inner-race temperatures are discussed.—(23.1).

POWER PLANTS

Performance calculations for a double-compound turbo-jet engine of 12:1 *design compressor pressure ratio. D. H. Mallinson and W. G. E. Lewis. R. & M.* 2645.

This report describes a theoretical investigation using conventional component characteristics to discover that division of work between the low and high-pressure compressors of a double-compound simple-jet gas turbine of 12:1 design pressure ratio which is likely to result in the most desirable equilibrium operation over the normal engine speed range. Having decided in favour of a pressure ratio of 3:1 in the low-pressure compressor and 4:1 in the other, a study is then made using more realistic compressor characteristics to determine the probable performance of such an engine under all flight conditions when the design maximum temperature is 900 deg. C (1,173 deg. K). The equilibrium running conditions of the engine are investigated with special reference to the problems introduced by the double-compound type of design.—(27.1).

RESEARCH

See also AIRCRAFT OPERATION—INTERNAL FLOW

Auxiliary equipment and techniques for adapting the constant-temperature hot-wire anemometer to specific problems in airflow measurements. J. C. Laurence and L. G. Landes. N.A.C.A. Technical Note 2843.

The constant-temperature hot-wire anemometer amplifier and accessories have been developed to provide an instrument with wide frequency response, good stability, and ease of operation. Auxiliary equipment has been developed to provide heating currents for large wires, to make average-square computations, and to make double-correlation coefficient measurements. Techniques are described for using this equipment to study periodic phenomena such as surge, rotating stall, and wake surveys in centrifugal- and axial-flow compressors. The application of the equipment to the study of non-periodic phenomena such as intensity, scale, and spectra of isotropic turbulence is also discussed.—(31.1).

Several combination probes for surveying static and total pressure and flow direction. W. M. Schulze, G. C. Ashby, Jr., and J. R. Erwin. N.A.C.A. Technical Note 2830.

An investigation has been conducted to provide a basis for design of combination probes intended to survey static and total pressure and direction of flow with special reference to subsonic turbo-machine testing. From the results of this investigation, the factors which determine the sensitivity of claw-type yaw probes were determined. Satisfactory combination survey probes for sensing static and total pressure and direction of flow in one or two planes were devised.—(31.1 × 1.5).

STRUCTURES

See also MATERIALS

An analysis of normal accelerations and airspeeds of one type of twin-engine transport airplane in commercial operations over a northern transcontinental route. R. Steiner. N.A.C.A. Technical Note 2833.

Normal acceleration and air speed data obtained for one type of twin-engine transport aeroplane in commercial operations over a northern trans-continental route are analysed to determine the gust and gust-load experiences of the aeroplane. The acceleration increments experienced equalled or exceeded the limit-gust-load factor, on the average, twice (once positive and once negative) in about $7{\cdot}5 \times 10^6$ flight miles, and an effective gust velocity of 30 ft./sec. was equalled or exceeded twice in about 1×10^6 flight miles.—(33.1.2).

Buckling of low arches or curved beams of small curvature. Y. C. Fung and A. Kaplan. N.A.C.A. Technical Note 2840.

A general solution, based on the classical buckling criterion, is given for the problem of buckling of low arches under a lateral loading acting toward the centre of curvature. The effects of initial end thrust and axial and lateral elastic support are discussed. The buckling load based on the energy criterion of Kármán and Tsien is also calculated. Results for both the classical and the energy criteria are compared with experimental results.—(33.2.1 × 33.2.4).

A theoretical and experimental investigation of the influence of temperature gradients on the deformation and burst speeds of rotating disks. P. I. Wilterdink, A. G. Holms and S. S. Manson. N.A.C.A. Technical Note 2803.

The purposes of this investigation were to evaluate the influence of temperature gradients and to test the validity of a recently developed method of calculating plastic flow in discs by comparing the calculated results with experimental observations. Short-time spin tests on parallel-sided, 10-inch-diameter discs were conducted under conditions that subjected the discs to a range of temperatures from 70° to 1,440°F. The agreement between the theoretical and experimental results was good over the range of temperature conditions investigated.—(33.2.2 × 33.2.4).

*Lateral instability of rectangular beams of strain hardening material under uniform bending. W. H. Wittrick. A.R.L. Australia. Report SM.*198.

The problem considered in this paper is that of the lateral buckling of initially straight beams of thin, deep rectangular cross-section under uniform bending, when the bending moment is large enough to cause plastic flow to occur over part of the cross-section. The solution applies to any shape of stress-strain curve. Numerical calculations are made for two different materials, one representative of an aluminium alloy, the other of annealed mild steel, and the two materials are compared.—(33.2.2).

*Evaluation of the theory on the post-buckling behaviour of stiffened, flat, rectangular plates subjected to shear and normal loads. W. K. G. Floor and T. J. Burgerhout. N.L.L. Amsterdam, Report S.*370.

The theory developed in N.L.L. Report S.295 has not been presented in such a form that it could be applied in a convenient way to the majority of practical problems. The numerical evaluation given covered only the special case of pure shear load acting upon the combination of plate and stiffeners. Additional numerical calculations were therefore made, the results of which have been presented in the form of a set of diagrams. Some numerical examples are given of the determination of the average stresses and strains or the required stiffener cross sections of specified stiffened plates subjected to specified compressive and shear loads. The case of relatively large compressive loads acting in the plane of the plate in the lateral direction could not fully be covered because the assumed waveform of the buckles in the plate proved to be inadequate in this range.—(33.2.4).

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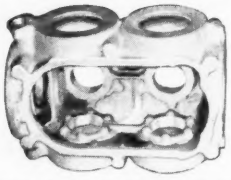

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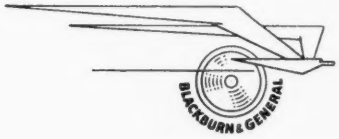

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